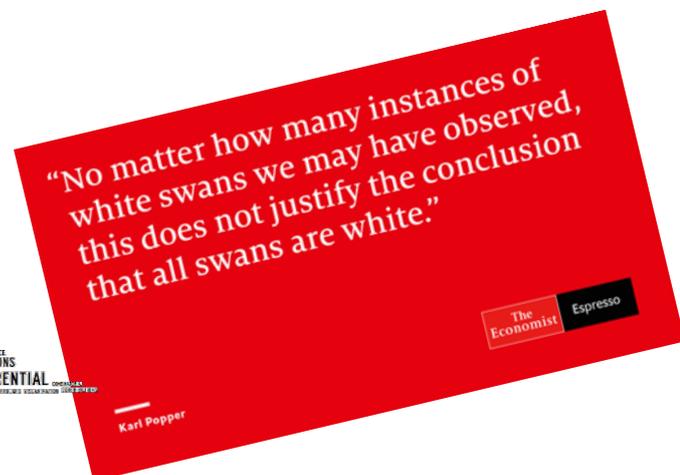
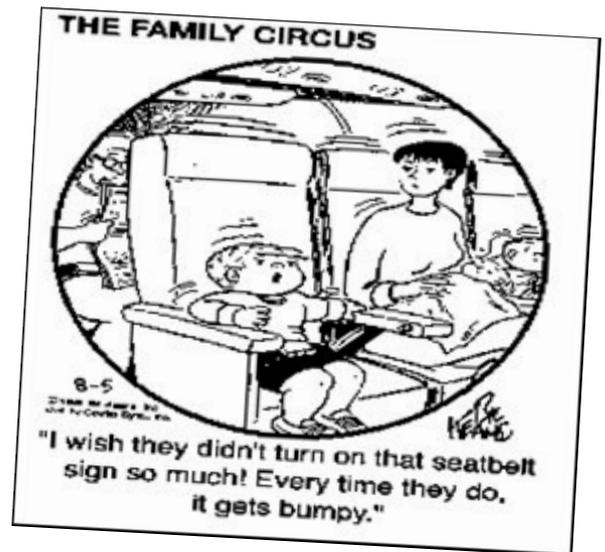


**Prof. Dr. Peter Schmidt**  
Summer Term 2019

# Quantitative Methods

## Executive MBA

- **Content**
- **References**
- **Material**



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Baye, M. “Managerial economics and business strategy“....” task “College Town” .....	68

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## Selected References

**Cooper**, Donald and **Schindler**, Pamela: “Business Research Methods”

**Koop**, Gary: “Analysis of Economic Data”

**Black**, Thomas: “Understanding Social Science Research“

**Baye**, Michael: “Managerial economics and business strategy”

**Blaxter**, Loraine; **Hughes**, Christian and **Tight**, Malcolm: "HOW TO Research"

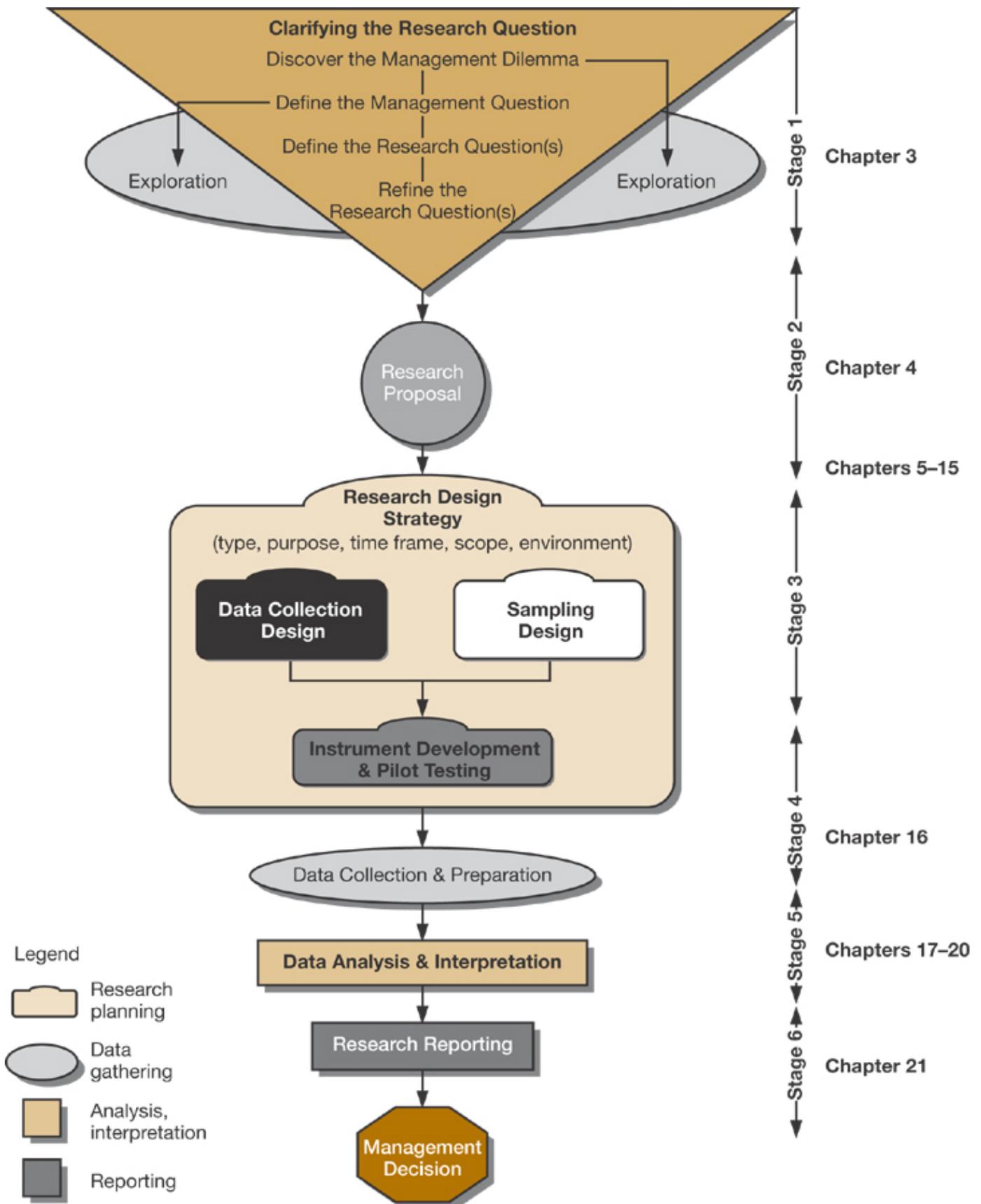
**Bryman**, Alan and **Cramer**, Duncan: “Quantitative data analysis with SPSS”

**Kinnear**, Paul and **Gray**, Colin: “SPSS made simple”

**Oakshott**, Les: “Essential Quantitative Methods for Business”

**Studenmund**, A. H.: “Using econometrics : a practical guide”

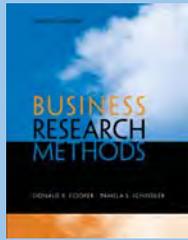
# The Business Research Process



Source: Cooper, D. and Schindler, P.S.: "Business Research Methods"

# Chapter 1

RESEARCH IN BUSINESS



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## Learning Objectives

### Understand . . .

- What business research is and how it differs from business decision support systems and business intelligence systems.
- Trends affecting business research and the emerging hierarchy of business decision makers.
- The distinction between good business research and research that falls short of professional quality.
- The nature of the research process.

## Pull Quote

“Forward-thinking executives recognize that analytics may be the only true source of sustainable advantage since it empowers employees at all levels of an organization with information to help them make smarter decisions.”

*Wayne Eckerson,  
director of research, business applications and  
architecture group,  
TechTarget*

## Why Study Business Research?

Business research provides information to guide business decisions

**Good research is like a parachute.**  
Without it, you could come to the wrong conclusion.

With research from JRP, you'll reach the right decision. For more than 40 years, we've worked with all agencies and corporate clients as partners, designing and leading projects of all types. See why our seasoned staff of project directors, interviewers, coders and analysts have led so many companies to come to the same conclusion: JRP. Call Paul Franzoni toll free at 877-JRP-2015 and ask about our full range of services.

**JRP**  
MARKETING RESEARCH

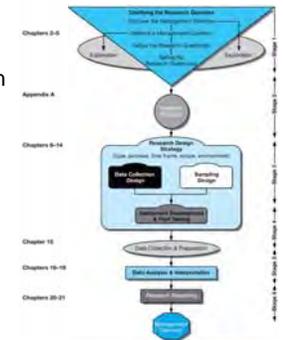
100 EAST WYOMING STREET, SUITE 100, P.O. BOX 1000  
DENVER, CO 80202-1000  
TEL: 303.733.1000 FAX: 303.733.1001  
WWW.JRP-ONLINE.COM

## Business Research

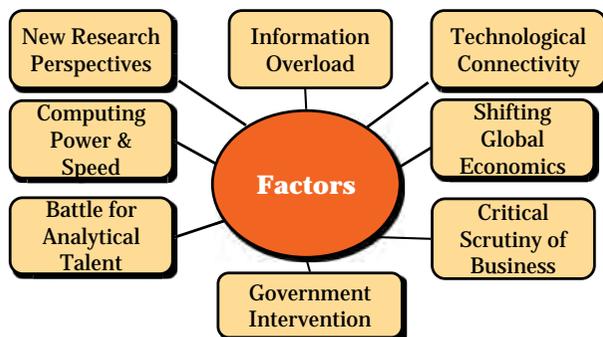
- A process of determining, acquiring, analyzing, synthesizing, and disseminating relevant business data, information, and insights to decision makers in ways that mobilize the organization to take appropriate business actions that, in turn, maximize business performance

## The Research Process

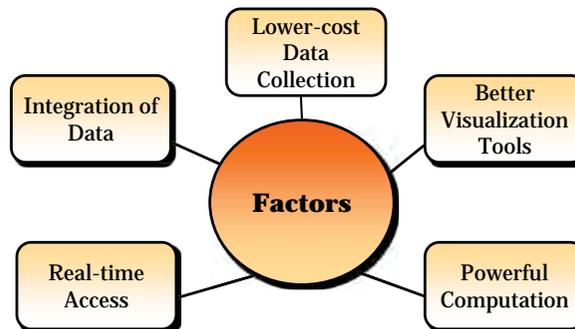
- Stage 1:** Clarifying the Research question
- Stage 2:** Proposing Research
- Stage 3:** Designing the Research
- Stage 4:** Data Collection & Preparation
- Stage 5:** Data Analysis & Interpretation
- Stage 6:** Reporting the Results



## What's Changing in Business that Influences Research



## Computing Power and Speed



## Information Sources

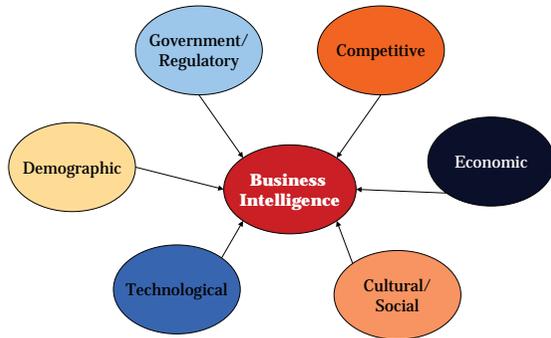
### Decision Support Systems

- Numerous elements of data organized for retrieval and use in business decision making
- Stored and retrieved via
  - Intranets
  - Extranets

### Business Intelligence Systems

- Ongoing information collection
- Focused on events, trends in micro and macro-environments

## Sources of Business Intelligence



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## Hierarchy of Business Decision Makers



11

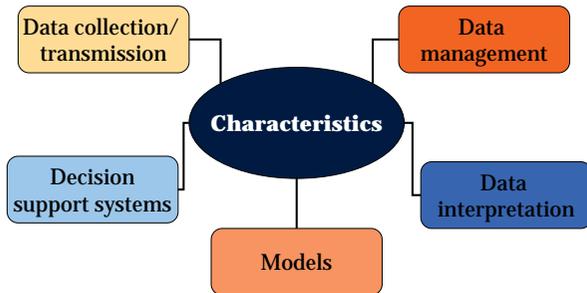
## Research May Not Be Necessary

### Can It Pass These Tests?

- Can information be applied to a critical decision?
- Will the information improve managerial decision making?
- Are sufficient resources available?

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## Information Value Chain



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## Chapter 2

### ETHICS IN BUSINESS RESEARCH



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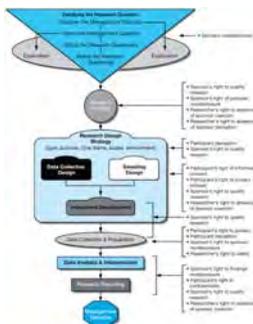
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## Ethical Treatment of Participants



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## Ethical Issues and the Research Process



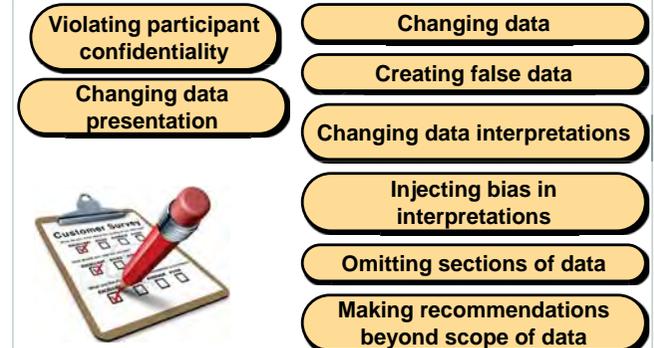
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## Types of Ethical Violations



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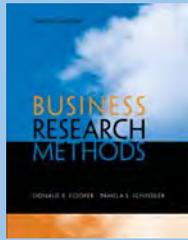
## Unethical Behavior to Avoid



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# Chapter 3

THINKING LIKE A RESEARCHER

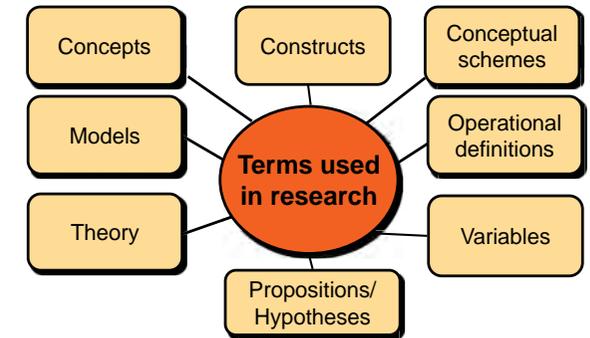


## Learning Objectives

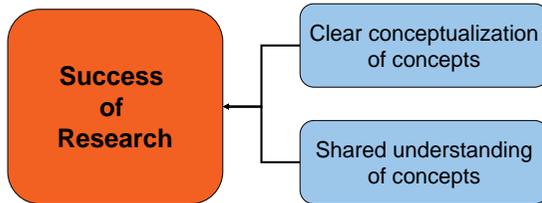
### Understand . . .

- The terminology used by professional researchers employing scientific thinking.
- What you need to formulate a solid research hypothesis.
- The need for sound reasoning to enhance research results.

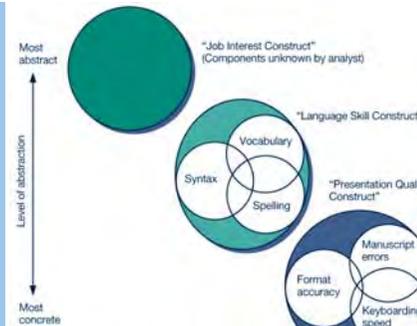
## Language of Research



## Language of Research



## Job Redesign Constructs and Concepts

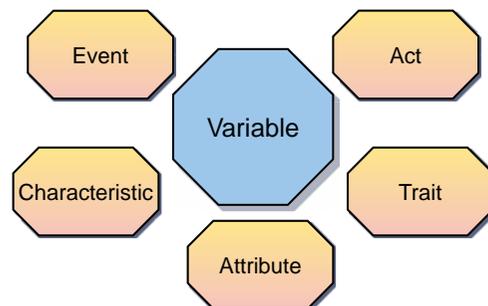


## Operational Definitions

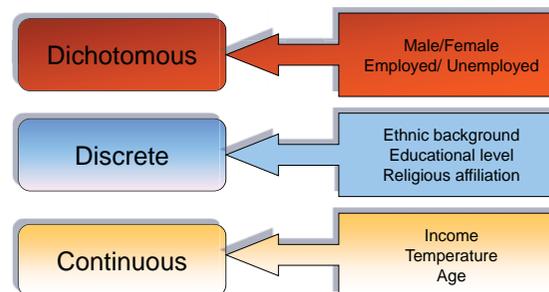
How can we define the variable "class level of students"?

- |             |                      |
|-------------|----------------------|
| • Freshman  | • < 30 credit hours  |
| • Sophomore | • 30-50 credit hours |
| • Junior    | • 60-89 credit hours |
| • Senior    | • > 90 credit hours  |

## A Variable: Property Being Studied



## Types of Variables



## Independent and Dependent Variable Synonyms

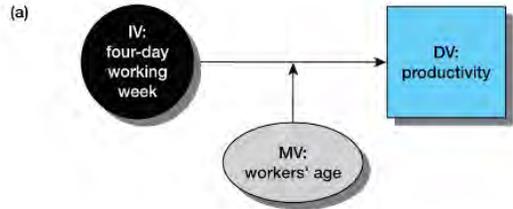
### Independent Variable (IV)

- Predictor
- Presumed cause
- Stimulus
- Predicted from...
- Antecedent
- Manipulated

### Dependent Variable (DV)

- Criterion
- Presumed effect
- Response
- Predicted to....
- Consequence
- Measured outcome

## Relationships Among Variable Types



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## Relationships Among Variable Types

### Moderating Variables (MV)

- The introduction of a four-day week (IV) will lead to higher productivity (DV), especially among younger workers (MV).
- The switch to commission from a salary compensation system (IV) will lead to increased sales (DV) per worker, especially more experienced workers (MV).
- The loss of mining jobs (IV) leads to acceptance of higher-risk behaviors to earn a family-supporting income (DV) - particularly among those with a limited education (MV).

### Extraneous Variables (EV)

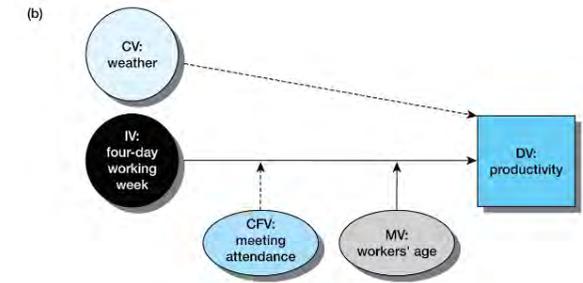
With new customers (EV-control), a switch to commission from a salary compensation system (IV) will lead to increased sales productivity (DV) per worker, especially among younger workers (MV).  
Among residents with less than a high school education (EV-control), the loss of jobs (IV) leads to high-risk behaviors (DV), especially due to the proximity of the firing range (MV).

### Intervening Variables (IVV)

- The switch to a commission compensation system (IV) will lead to higher sales (DV) by increasing overall compensation (IVV).
- A promotion campaign (IV) will increase savings activity (DV), especially when free prizes are offered (MV), but chiefly among smaller savers (EV-control). The results come from enhancing the motivation to save (IVV).

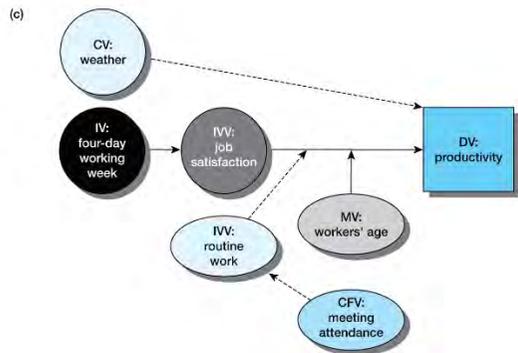
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## Relationships Among Variable Types



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## Relationships Among Variable Types



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## Descriptive Hypothesis Formats

### Descriptive Hypothesis

- In Detroit, our potato chip market share stands at 13.7%.
- American cities are experiencing budget difficulties.

### Research Question

- What is the market share for our potato chips in Detroit?
- Are American cities experiencing budget difficulties?

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## Relational Hypotheses Formats

### Correlational

- Young women (under 35) purchase fewer units of our product than women who are older than 35.
- The number of suits sold varies directly with the level of the business cycle.

### Causal

- An increase in family income leads to an increase in the percentage of income saved.
- Loyalty to a grocery store increases the probability of purchasing that store's private brand products.

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## The Role of Hypotheses

Guide the direction of the study

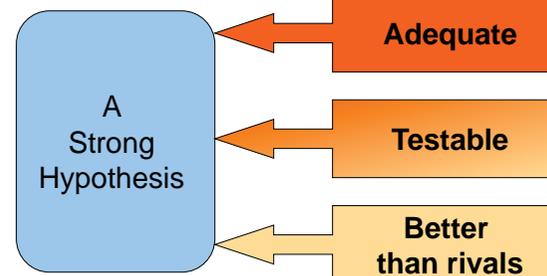
Identify relevant facts

Suggest most appropriate research design

Provide framework for organizing resulting conclusions

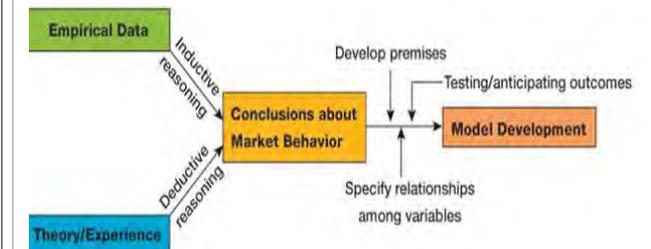
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## Characteristics of Strong Hypotheses



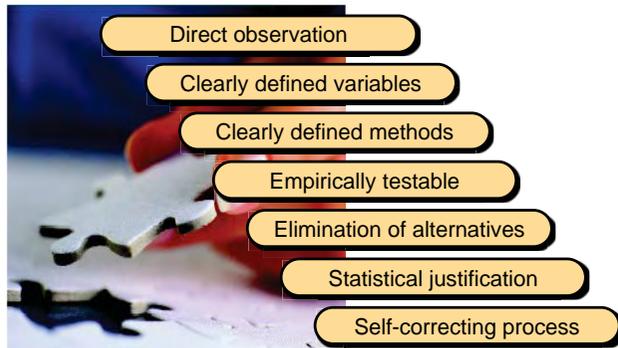
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## The Role of Reasoning



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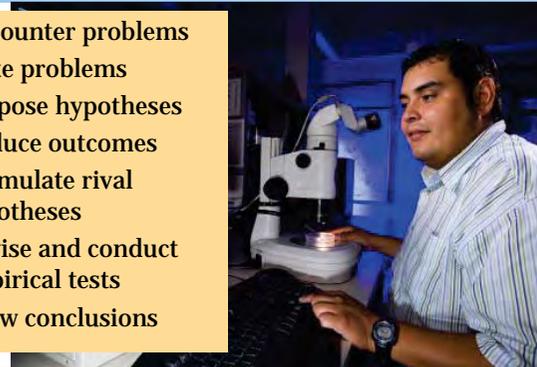
## The Scientific Method



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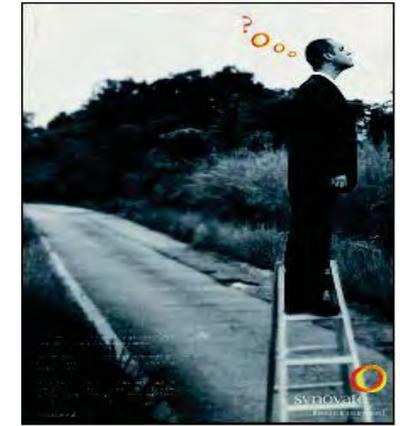
## Researchers

- Encounter problems
- State problems
- Propose hypotheses
- Deduce outcomes
- Formulate rival hypotheses
- Devise and conduct empirical tests
- Draw conclusions



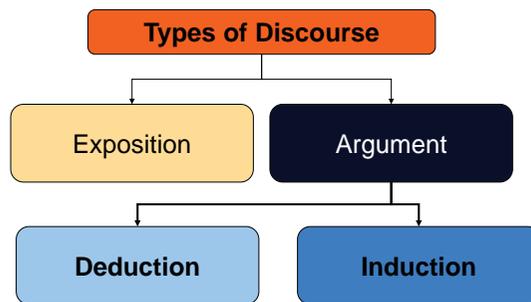
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## Why is curiosity important?



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## Sound Reasoning

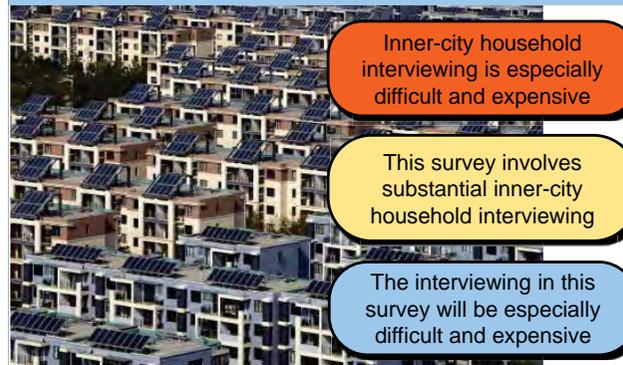


Deduction is a form of reasoning in which the conclusion must necessarily follow from the premises given.

Induction is a form of reasoning that draws a conclusion from one or more particular facts or pieces of evidence.

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## Deductive Reasoning



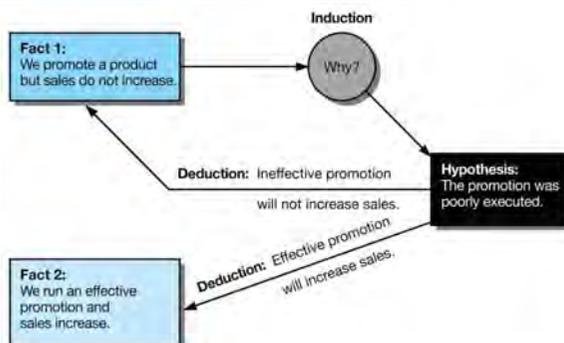
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## Inductive Reasoning

- **Why didn't sales increase during our promotional event?**
  - Regional retailers did not have sufficient stock to fill customer requests during the promotional period
  - A strike by employees prevented stock from arriving in time for promotion to be effective
  - A hurricane closed retail outlets in the region for 10 days during the promotion

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## Why Didn't Sales Increase?



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## Chapter 4

### THE RESEARCH PROCESS: AN OVERVIEW



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## Learning Objectives

### Understand ...

- That research is decision- and dilemma-centered.
- That the clarified research question is the result of careful exploration and analysis and sets the direction for the research project.

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## Learning Objectives

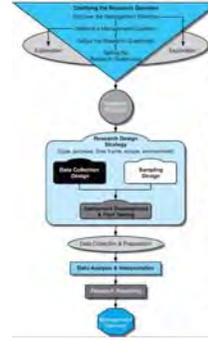
### Understand . . .

- How value assessments and budgeting influence the process for proposing research, and ultimately, research design.
- What is included in research design, data collection, and data analysis.
- Research process problems to avoid.

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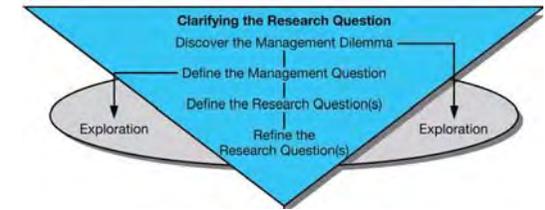
## The Research Process

- Stage 1:** Clarifying the Research question
- Stage 2:** Proposing Research
- Stage 3:** Designing the Research
- Stage 4:** Data Collection & Preparation
- Stage 5:** Data Analysis & Interpretation
- Stage 6:** Reporting the Results



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## Stage 1: Clarifying the Research Question

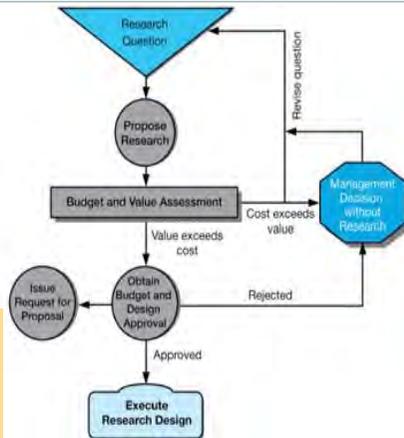


Management-research question hierarchy process begins by identifying the management dilemma

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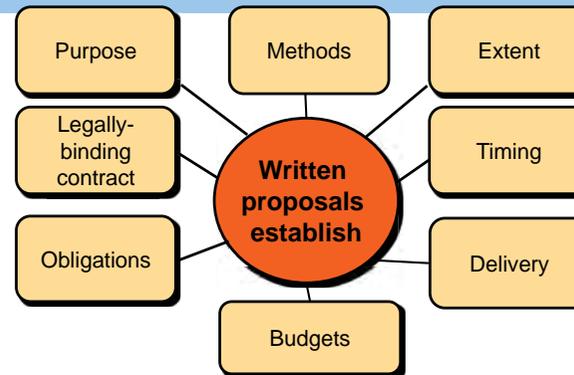
## Stage 2: Proposing Research

- **Budget Types**
  - Rule-of-thumb
  - Departmental
  - Task



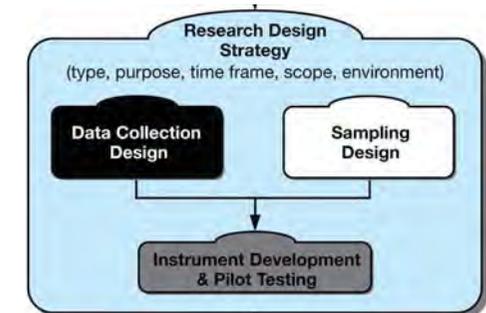
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## The Research Proposal



50

## Stage 3: Designing the Research



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## Stage 4: Data Collection

Data Collection & Preparation

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## Data Characteristics

Abstractness

Verifiability

Elusiveness

Closeness

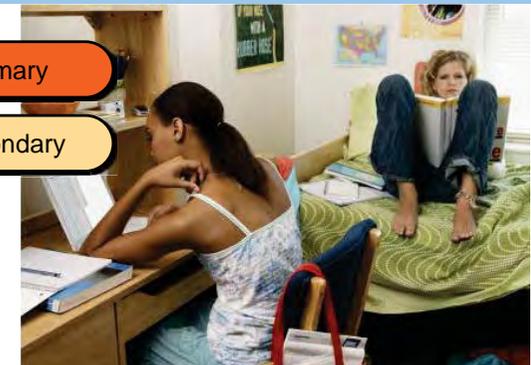


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## Data Types

Primary

Secondary



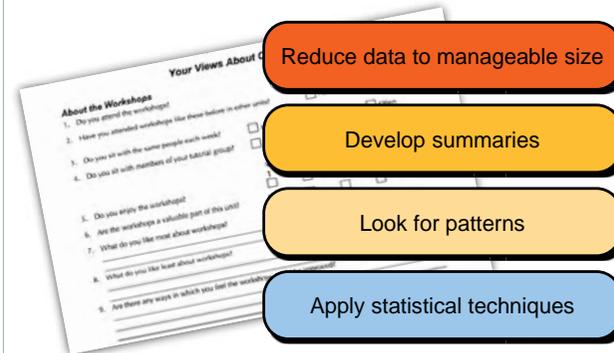
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## Stage 5: Data Analysis & Interpretation

Data Analysis & Interpretation

55

## Steps in Data Analysis and Interpretation



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## Stage 6: Reporting the Results

Research Reporting

Management Decision

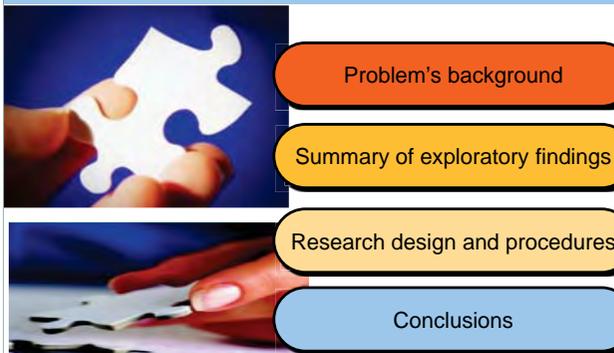
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## Parts of the Research Report



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## The Research Report Overview



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## Chapter 6

RESEARCH DESIGN: AN OVERVIEW



McGraw-Hill Irwin

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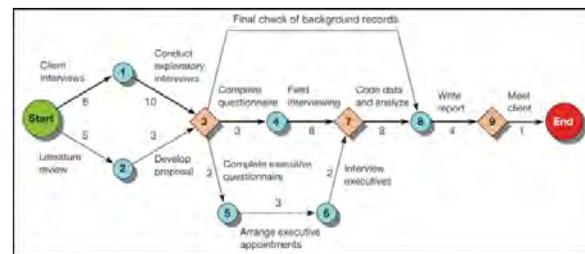
## Learning Objectives

*Understand . . .*

- The basic stages of research design.
- The major descriptors of research design.
- The major types of research designs.
- The relationships that exist between variables in research design and the steps for evaluating those relationships.

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## What Tools Are Used in Designing Research?



**Milestones:**  
 3 Proposal approval  
 7 Interviews completed  
 9 Final report completed

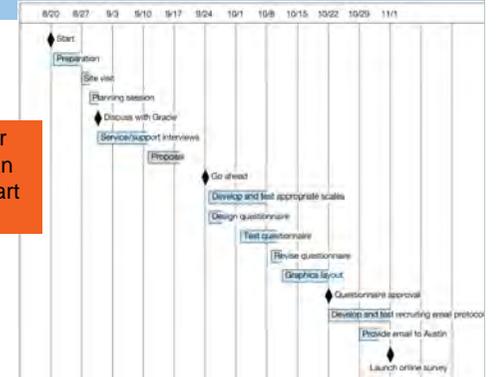
**Critical Path:**  
 5-1-3-4-7-8-9-E

**Time to Completion:**  
 40 working days

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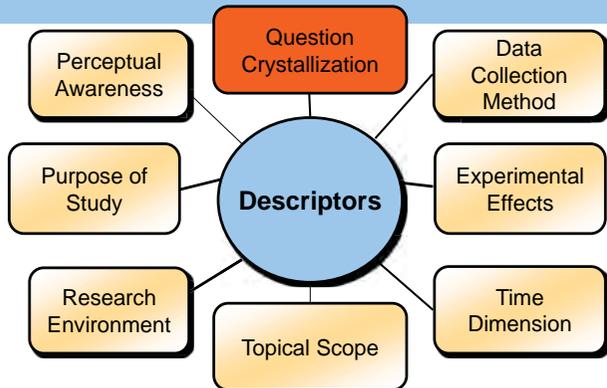
## What Tools Are Used in Designing Research?

MindWriter  
Project Plan  
in Gantt chart  
format



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## Research Design Descriptors



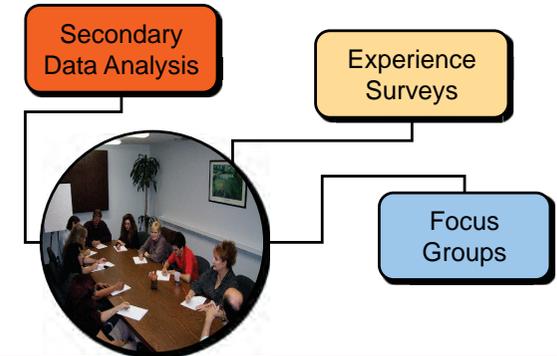
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## Degree of Question Crystallization

- **Exploratory Study**
  - Loose structure
  - Expand understanding
  - Provide insight
  - Develop hypotheses
- **Formal Study**
  - Precise procedures
  - Begins with hypotheses
  - Answers research questions

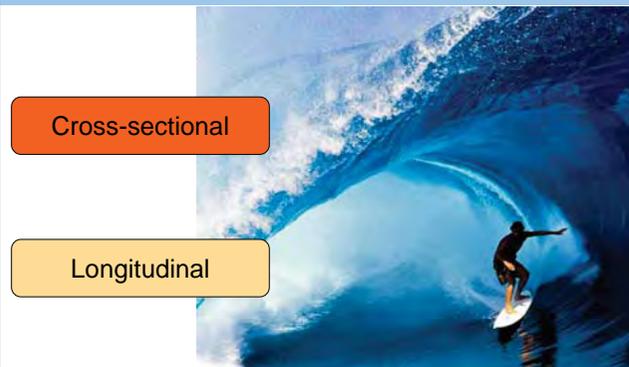
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## Commonly Used Exploratory Techniques



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## The Time Dimension



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## The Research Environment



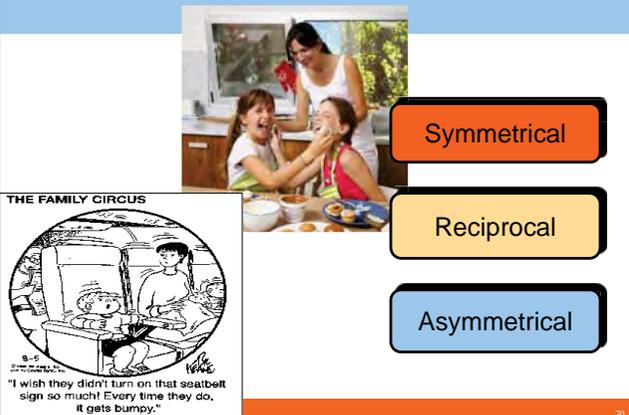
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## Descriptive Studies



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## Causal Studies



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## Understanding Casual Relationships



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## Chapter 10

SURVEYS



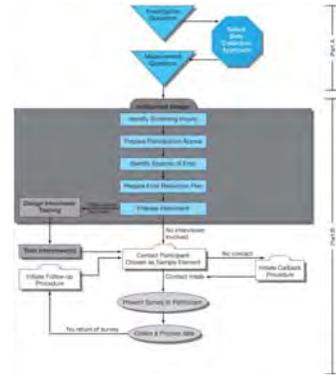
## Learning Objectives

### Understand . . .

- The process for selecting the appropriate and optimal communication approach.
- Factors affect participation in communication studies.
- Sources of error in communication studies and how to minimize them.
- Major advantages and disadvantages of the three communication approaches.
- Why an organization might outsource a communication study.

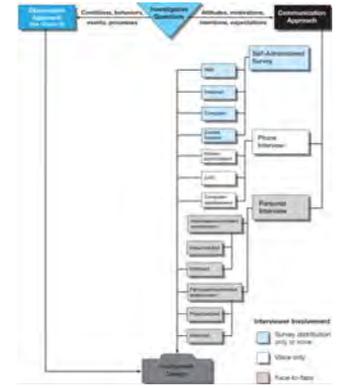
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## Data Collection Approach



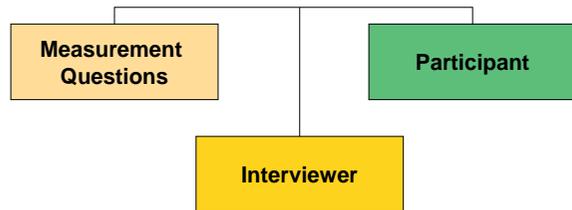
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## Selecting a Communication Data Collection Approach



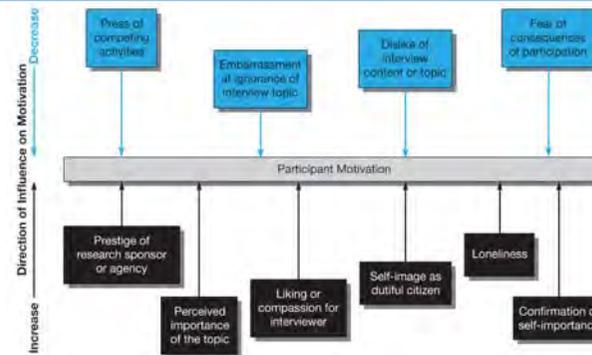
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## Sources of Error



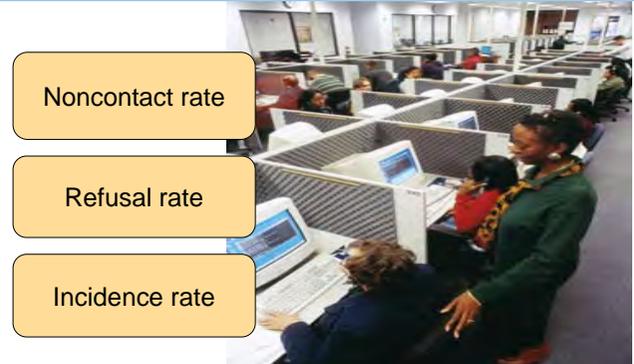
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## Participant Motivation



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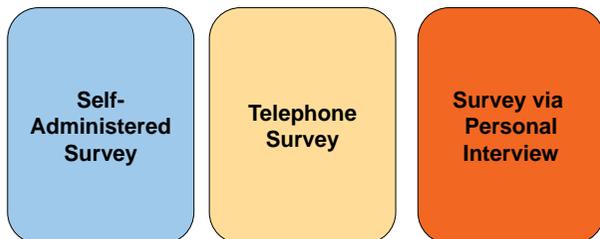
## Response Terms



- Noncontact rate
- Refusal rate
- Incidence rate

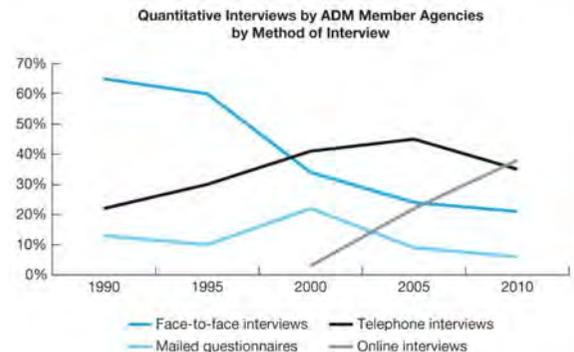
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## Communication Approaches



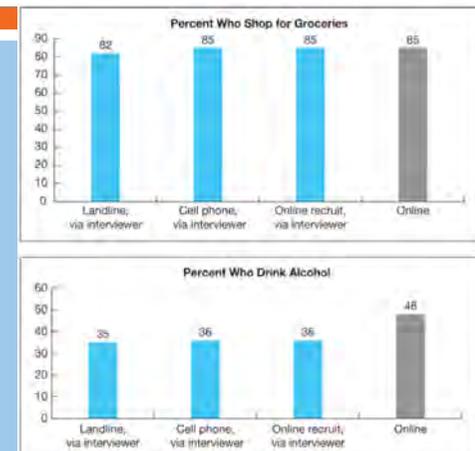
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## PicProfile: Methodology Trends



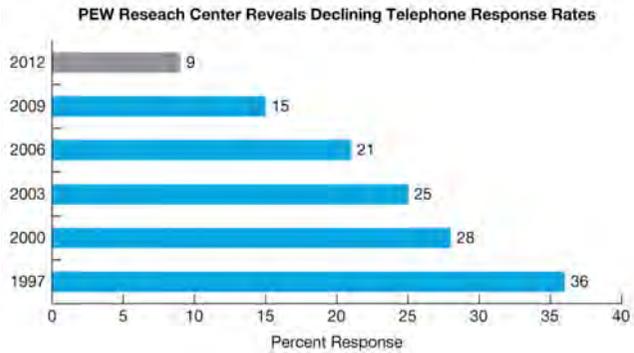
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## PicProfile: Mixed-mode Research



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## PicProfile: Declining Phone Response



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## Chapter 11

MEASUREMENT



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## Learning Objectives

### Understand . . .

- The distinction between measuring objects, properties, and indicants of properties.
- The similarities and differences between the four scale types used in measurement and when each is used.
- The four major sources of measurement error.
- The criteria for evaluating good measurement.

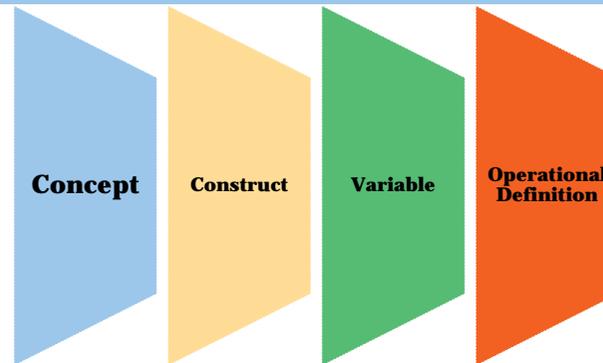
## Pull Quote

“You’re trying too hard to find a correlation here. You don’t know these people, you don’t know what they intended. You try to compile statistics and correlate them to a result that amounts to nothing more than speculation.”

*Marc Racicot, former governor of Montana and chairman of the Republican Party*

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## Review of Terms



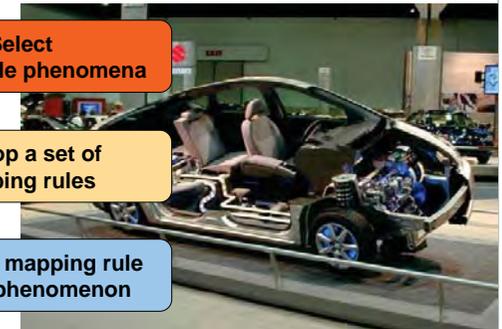
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## Measurement

Select measurable phenomena

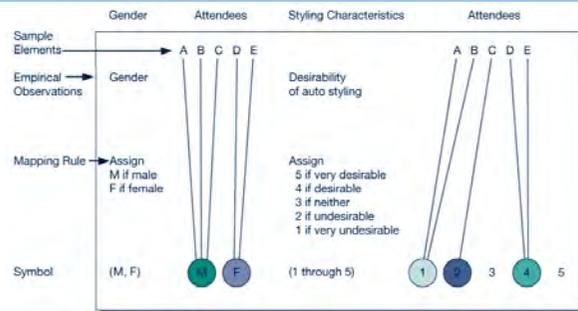
Develop a set of mapping rules

Apply the mapping rule to each phenomenon



87

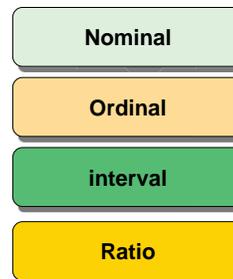
## Characteristics of Measurement



Attendees A, B, and C are male, and find the auto's styling to be undesirable.  
Attendees D and E are female and find the auto's styling desirable.

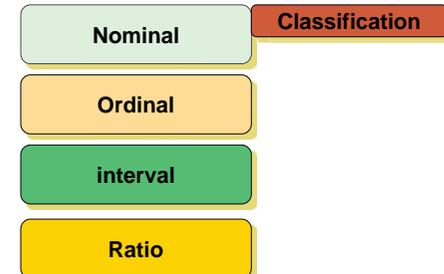
88

## Types of Scales



89

## Levels of Measurement



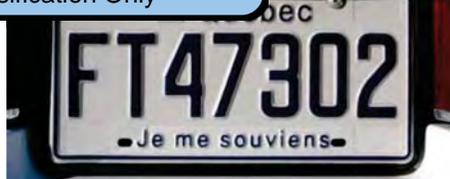
90

## Nominal Scales

Mutually Exclusive

Collectively Exhaustive Categories

Classification Only



91

## Levels of Measurement

Nominal

Classification

Ordinal

Classification

Order

Interval

Ratio

92

## Ordinal Scales



Nominal Scale Characteristics

Order

Implies greater than or less than

93

## Levels of Measurement

Nominal

Classification

Ordinal

Classification

Order

Interval

Classification

Distance

Order

Ratio

94

## Interval Scales

Ordinal Scale Characteristics

Equality of interval

Equality of distance between numbers



95

## Levels of Measurement

Nominal

Classification

Ordinal

Classification

Order

Interval

Classification

Distance

Order

Ratio

Classification

Distance

Order

Natural Origin

96

## Ratio Scales

Interval Scale Characteristics

Absolute Zero



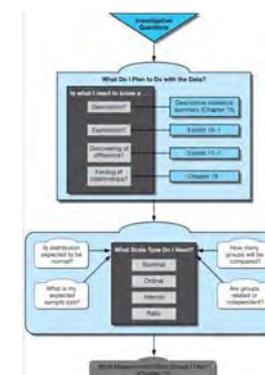
97

## Examples of Data Scales

Type of Scale	Example
Nominal	Gender (male, female)
Ordinal	Doneness of meat (well, medium well, medium rare, rare)
Interval	Temperature in degrees
Ratio	Age in years

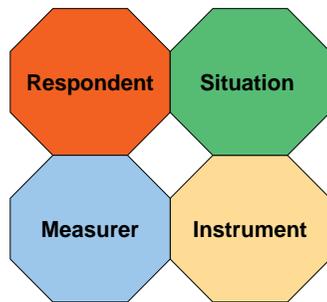
98

## From Investigative to Measurement Questions



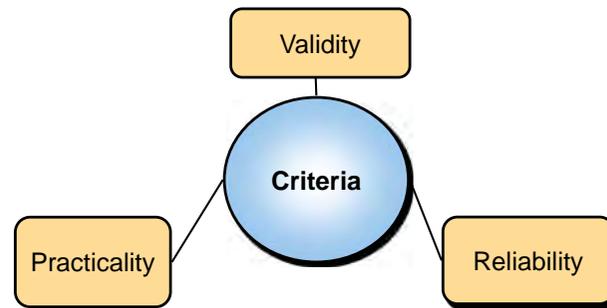
99

## Sources of Error



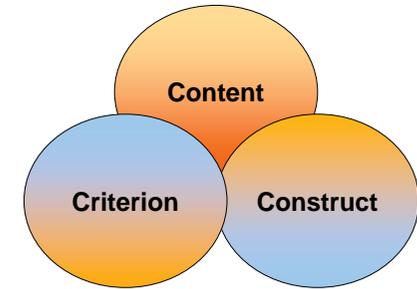
100

## Evaluating Measurement Tools



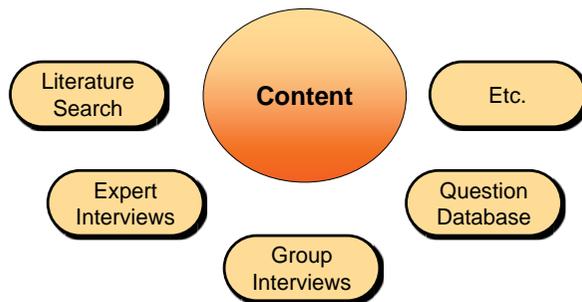
101

## Validity Determinants



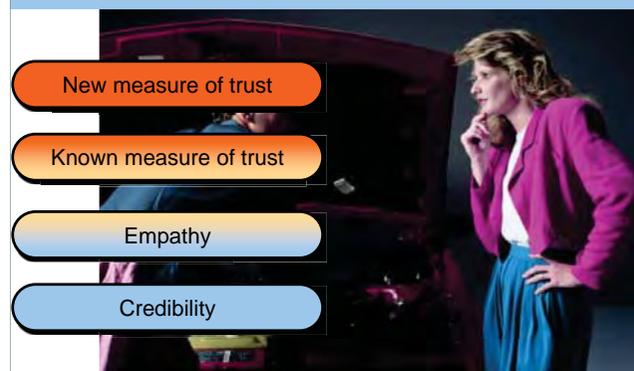
102

## Increasing Content Validity



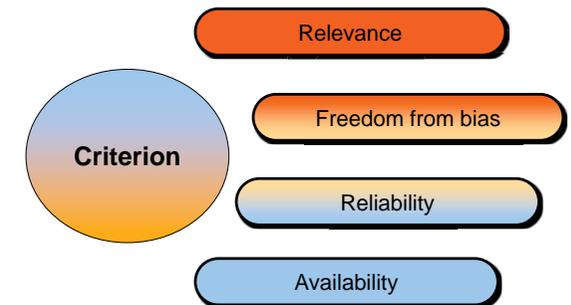
103

## Increasing Construct Validity



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## Judging Criterion Validity



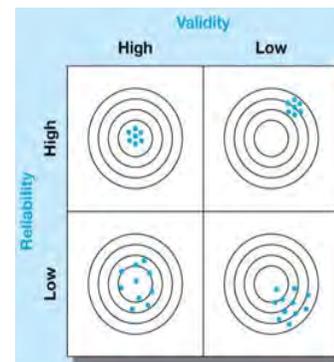
105

## Summary of Validity Estimates

Types	What is Measured	Methods
<b>Content</b>	Degree to which the content of the items actually represents the universe of all relevant items under study.	<ul style="list-style-type: none"> <li>Judgmental</li> <li>Panel evaluation with content validity ratio</li> </ul>
<b>Criterion-Related</b>	Degree to which the predictor is adequate in capturing the relevant aspects of the criterion.	<ul style="list-style-type: none"> <li>Correlation</li> </ul>
<b>Concurrent</b>	Description of the present; criterion data are available at the same time as predictor scores.	<ul style="list-style-type: none"> <li>Correlation</li> </ul>
<b>Predictive</b>	Prediction of the future; criterion data are measured after the passage of time.	<ul style="list-style-type: none"> <li>Correlation</li> </ul>
<b>Construct</b>	Answers the question, "What accounts for the variance in the measure?"; attempts to identify the underlying construct(s) being measured and determine how well the test represents it (them).	<ul style="list-style-type: none"> <li>Judgmental</li> <li>Correlation of proposed test with established one</li> <li>Convergent-discriminant techniques</li> <li>Factor analysis</li> <li>Multitrait-multimethod analysis</li> </ul>

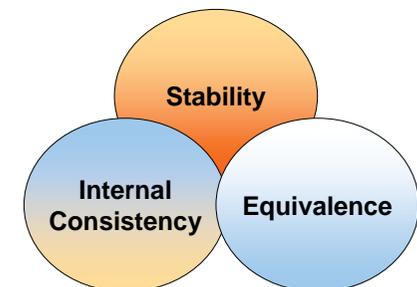
106

## Understanding Validity and Reliability



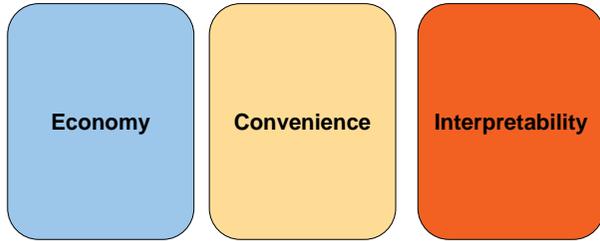
107

## Reliability Estimates



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## Practicality



109

## Chapter 12

### MEASUREMENT SCALES



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## Learning Objectives

### Understand...

- The nature of attitudes and their relationship to behavior.
- The critical decisions involved in selecting an appropriate measurement scale.
- The characteristics and use of rating, ranking, sorting, and other preference scales.

## Pull Quote

“No man learns to know his inmost nature by introspection, for he rates himself sometimes too low, and often too high, by his own measurement. Man knows himself only by comparing himself with other men; it is life that touches his genuine worth.”

*Johann Wolfgang von Goethe*  
German writer, artist, politician  
(1749–1832)

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## Nature of Attitudes

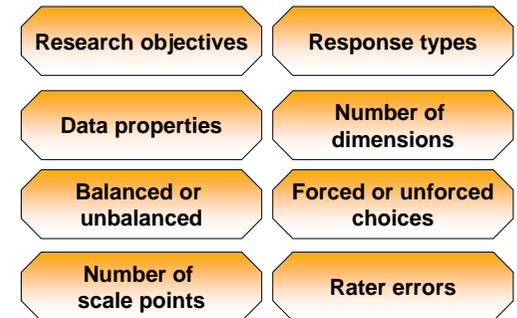
**Cognitive**      I think oatmeal is healthier than corn flakes for breakfast.

**Affective**      I hate corn flakes.

**Behavioral**      I intend to eat more oatmeal for breakfast.

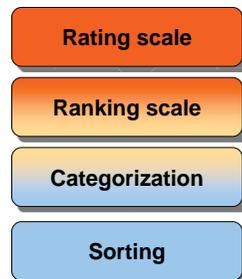
113

## Selecting a Measurement Scale



114

## Response Types



115

## Number of Dimensions

Unidimensional

Multi-dimensional

116

## Balanced or Unbalanced

How good an actress is Jennifer Lawrence?

Very bad  
Bad  
Neither good nor bad  
Good  
Very good

Poor  
Fair  
Good  
Very good  
Excellent

117

## Forced or Unforced Choices

How good an actress is Jennifer Lawrence?

Very bad  
Bad  
Neither good nor bad  
Good  
Very good

Very bad  
Bad  
Neither good nor bad  
Good  
Very good  
No opinion  
Don't know

118

## Number of Scale Points

How good an actress is Jennifer Lawrence?

Very bad  
Bad  
Neither good nor bad  
Good  
Very good

Very bad  
Somewhat bad  
A little bad  
Neither good nor bad  
A little good  
Somewhat good  
Very good

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## Rater Errors

Error of central tendency  
Error of leniency

- Adjust strength of descriptive adjectives
- Space intermediate descriptive phrases farther apart
- Provide smaller differences in meaning between terms near the ends of the scale
- Use more scale points

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## Rater Errors

Primacy Effect  
Recency Effect

Reverse order of alternatives periodically or randomly

121

## Rater Errors

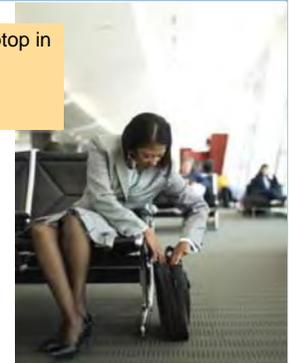
Halo Effect

- Rate one trait at a time
- Reveal one trait per page
- Reverse anchors periodically

122

## Simple Category Scale

I plan to purchase a MindWriter laptop in the next 12 months.  
 Yes  
 No

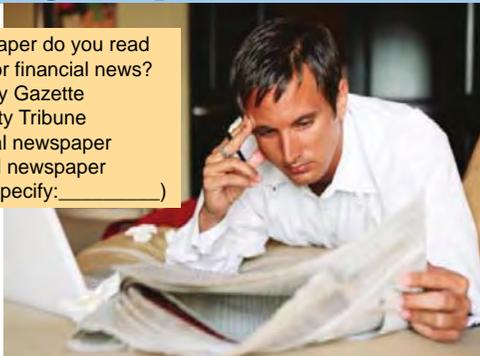


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## Multiple-Choice, Single-Response Scale

What newspaper do you read most often for financial news?

- East City Gazette
- West City Tribune
- Regional newspaper
- National newspaper
- Other (specify: \_\_\_\_\_)



124

## Multiple-Choice, Multiple-Response Scale

Check any of the sources you consulted when designing your new home.

- Online planning services
- Magazines
- Independent contractor/builder
- Designer
- Architect
- Other (specify: \_\_\_\_\_)

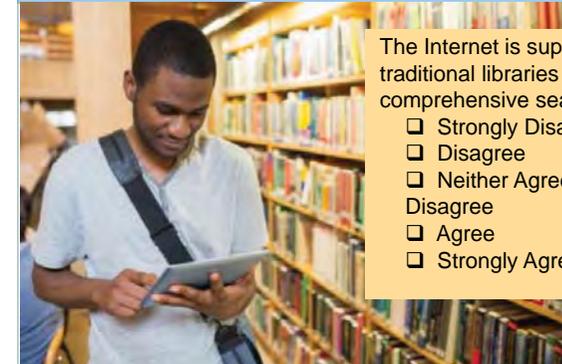


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## Likert Scale

The Internet is superior to traditional libraries for comprehensive searches.

- Strongly Disagree
- Disagree
- Neither Agree nor Disagree
- Agree
- Strongly Agree



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# Semantic Differential



Lands' End Catalog  
 FAST \_\_\_\_\_ SLOW  
 HIGH QUALITY \_\_\_\_\_ LOW QUALITY

# Adapting SD Scales

**Convenience of Reaching the Store from Your Location**

Nearby \_\_\_\_\_ Distant \_\_\_\_\_  
 Short time required to reach store \_\_\_\_\_ Long time required to reach store \_\_\_\_\_  
 Difficult drive \_\_\_\_\_ Easy Drive \_\_\_\_\_  
 Difficult to find parking place \_\_\_\_\_ Easy to find parking place \_\_\_\_\_  
 Convenient to other stores / shop \_\_\_\_\_ Inconvenient to other stores / shop \_\_\_\_\_

**Products offered**

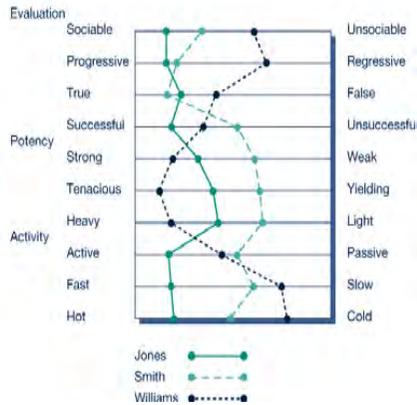
Wide selection of different kinds of products \_\_\_\_\_ Limited selection of different kinds of products \_\_\_\_\_  
 Fully stocked \_\_\_\_\_ Understocked \_\_\_\_\_  
 Undependable products \_\_\_\_\_ Dependable products \_\_\_\_\_  
 High quality \_\_\_\_\_ Low quality \_\_\_\_\_  
 Numerous brands \_\_\_\_\_ Few brands \_\_\_\_\_  
 Unknown brands \_\_\_\_\_ Well-known brands \_\_\_\_\_

# SD Scale for Analyzing Actor Candidates

Analyze (candidate) for current position:

(E) Sociable	(7): _____	(1) Unsociable
(P) Weak	(1): _____	(7) Strong
(A) Active	(7): _____	(1) Passive
(E) Progressive	(7): _____	(1) Regressive
(P) Yielding	(1): _____	(7) Tenacious
(E) Slow	(1): _____	(7) Fast
(E) True	(7): _____	(1) False
(P) Heavy	(7): _____	(1) Light
(A) Hot	(7): _____	(1) Cold
(E) Unsuccessful	(1): _____	(7) Successful

# Graphic of SD Analysis



# Numerical Scale

EXTREMELY FAVORABLE 5 4 3 2 1 EXTREMELY UNFAVORABLE

Employee's cooperation in teams \_\_\_\_\_  
 Employee's knowledge of task \_\_\_\_\_  
 Employee's planning effectiveness \_\_\_\_\_

# Multiple Rating List Scales

"Please indicate how important or unimportant each service characteristic is:"

	IMPORTANT						UNIMPORTANT
Fast, reliable repair	7	6	5	4	3	2	1
Service at my location	7	6	5	4	3	2	1
Maintenance by manufacturer	7	6	5	4	3	2	1
Knowledgeable technicians	7	6	5	4	3	2	1
Notification of upgrades	7	6	5	4	3	2	1
Service contract after warranty	7	6	5	4	3	2	1



# Stapel Scales

	(Company Name)	
+5	+5	+5
+4	+4	+4
+3	+3	+3
+2	+2	+2
+1	+1	+1
Technology Leader	Exciting Products	World-Class Reputation
-1	-1	-1
-2	-2	-2
-3	-3	-3
-4	-4	-4
-5	-5	-5

# Constant-Sum Scales

"Taking all the supplier characteristics we've just discussed and now considering cost, what is their relative importance to you (dividing 100 units between)?"

Being one of the lowest-cost suppliers

All other aspects of supplier performance

Sum 100

# Graphic Rating Scales

MindWriter

"How likely are you to recommend CompleteCare to others?" (Place an X at the position along the line that best reflects your judgment.)

VERY LIKELY  VERY UNLIKELY

(alternative with graphic)

## Ranking Scales



Paired-comparison scale

Forced ranking scale

Comparative scale

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## Paired-Comparison Scale

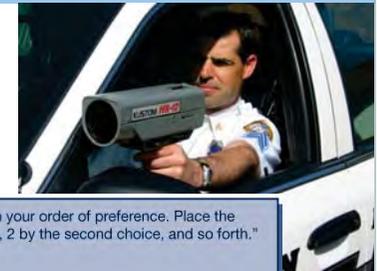


"For each pair of two-seat sports cars listed, place a check beside the one you would most prefer if you had to choose between the two."

- |   |   |
|---|---|
| <input type="checkbox"/> BMW Z4 M Coupe         | <input type="checkbox"/> Chevrolet Corvette Z06 |
| <input type="checkbox"/> Porsche Cayman S       | <input type="checkbox"/> Porsche Cayman S       |
| <input type="checkbox"/> Chevrolet Corvette Z06 | <input type="checkbox"/> Porsche Cayman S       |
| <input type="checkbox"/> BMW Z4 M Coupe         | <input type="checkbox"/> Dodge Viper SRT10      |
| <input type="checkbox"/> Chevrolet Corvette Z06 | <input type="checkbox"/> Dodge Viper SRT10      |
| <input type="checkbox"/> Dodge Viper SRT10      | <input type="checkbox"/> BMW Z4 M Coupe         |

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## Forced Ranking Scale



"Rank the radar detection features in your order of preference. Place the number 1 next to the most preferred, 2 by the second choice, and so forth."

- User programming
- Cordless capability
- Small size
- Long-range warning
- Minimal false alarms

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## Comparative Scale

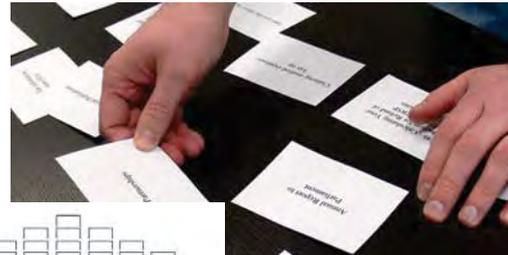


"Compared to your previous hair dryer's performance, the new one is:"

- |                            |  |                            |
|----------------------------|--|----------------------------|
| SUPERIOR                   | ABOUT THE SAME   | INFERIOR                   |
| <input type="checkbox"/> 1 | <input type="checkbox"/> 2 <input type="checkbox"/> 3 <input type="checkbox"/> 4 | <input type="checkbox"/> 5 |

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## Sorting



Example of a Q-Sort

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## CloseUp: MindWriter Scaling



Likert Scale				
The problem that prompted service/repair was resolved				
Strongly Disagree	Disagree	Neither Agree Nor Disagree	Agree	Strongly Agree
1	2	3	4	5

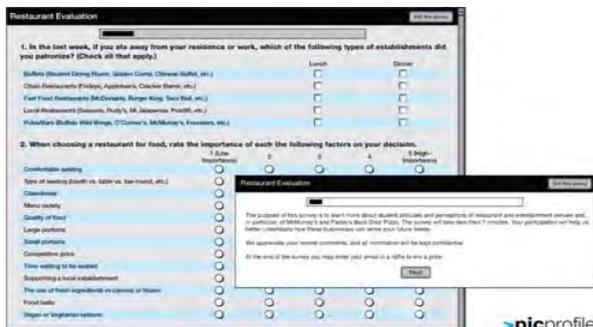
Numerical Scale (MindWriter's Favorite)				
To what extent are you satisfied that the problem that prompted service/repair was resolved?				
Very Dissatisfied				Very Satisfied
1	2	3	4	5

Hybrid Expectation Scale				
Resolution of the problem that prompted service/repair.				
Met Few Expectations	Met Some Expectations	Met Most Expectations	Met All Expectations	Exceeded Expectations
1	2	3	4	5

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## PicProfile: Online Surveys



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## Chapter 13

### QUESTIONNAIRES AND INSTRUMENTS



McGraw-Hill/Irwin

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## Learning Objectives

### Understand...

- The link forged between the management dilemma and the communication instrument by the management-research question hierarchy.
- The influence of the communication method on instrument design.
- The three general classes of information and what each contributes to the instrument.

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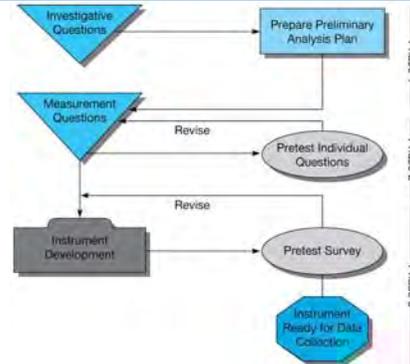
## Learning Objectives

### Understand . . .

- The influence of question content, question wording, response strategy, and preliminary analysis planning on question construction.
- Each of the numerous question design issues influencing instrument quality, reliability, and validity.
- The sources for measurement questions
- The importance of pretesting questions and instruments.

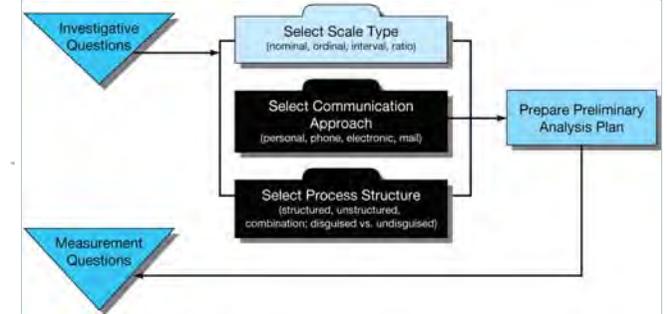
145

## Overall Flowchart for Instrument Design



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## Flowchart for Instrument Design Phase 1



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## Strategic Concerns in Instrument Design

What type of scale is needed?

What communication approach will be used?

Should the questions be structured?

Should the questioning be disguised?

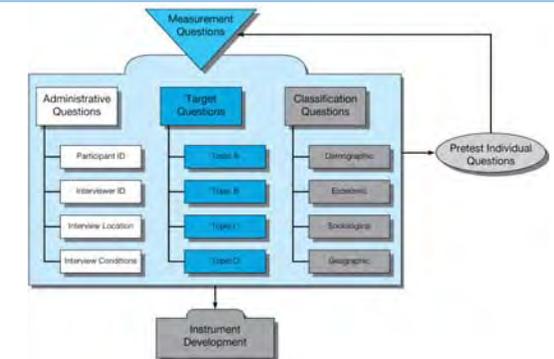
148

## Factors Affecting Respondent Honesty

Syndrome	Description	Example
Peacock	Desire to be perceived as smarter, wealthier, happier, or better than others.	Respondent who claims to shop Harrods in London twice as many as those that do.
Pleaser	Desire to help by providing answers they think the researchers want to hear, to please or avoid offending or being socially stigmatized.	Respondent gives a politically correct or assured correct answer about degree to which they revere their elders, respect their spouse, etc.
Gamer	Adaption of answers to play the system.	Participants who take membership to a specific demographic to participate in high remuneration study; that they drive an expensive car when they don't or that they have cancer when they don't.
Disengager	Don't want to think deeply about a subject.	Falsify ad recall or purchase behavior (didn't recall or didn't buy) when they actually did.
Self-delusionist	Participants who lie to themselves.	Respondent who tallies behavior, like the level they recycle.
Unconscious Decision Maker	Participants who are dominated by irrational decision making.	Respondent who cannot predict with any certainty his future behavior.
Ignoramus	Participant who never knew or doesn't remember an answer and makes up a lie.	Respondent who can't identify on a map where they live or remember what they ate for supper the previous evening.

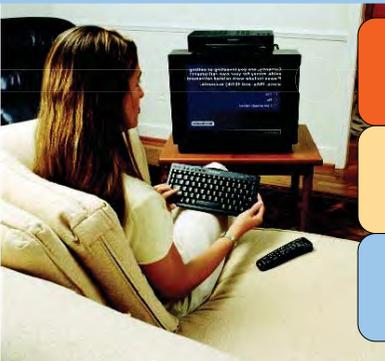
149

## Flowchart for Instrument Design Phase 2



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## Question Categories and Structure



Administrative

Target

Classification

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## Engagement = Convenience

*"Participants are becoming more and more aware of the value of their time. The key to maintaining a quality dialog with them is to make it really convenient for them to engage, whenever and wherever they want."*

Tom Anderson  
managing partner  
Anderson Analytics

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## Question Content

Should this question be asked?

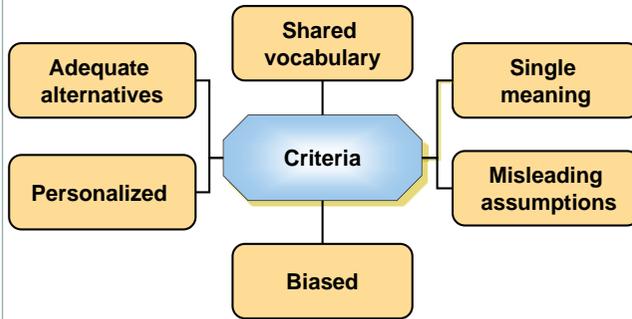
Is the question of proper scope and coverage?

Can the participant adequately answer this question as asked?

Will the participant willingly answer this question as asked?

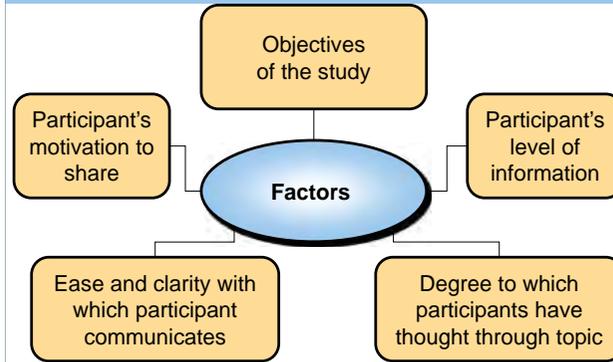
153

## Question Wording



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## Response Strategy



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## Free-Response Strategy



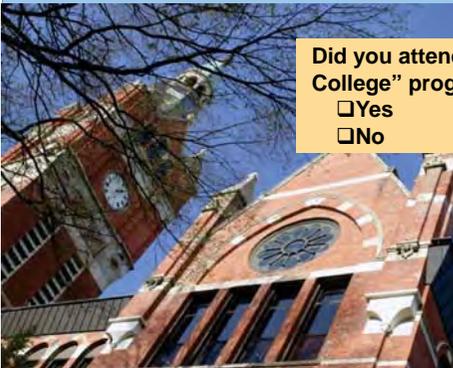
What factors influenced your enrollment in Metro U?

\_\_\_\_\_

\_\_\_\_\_

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## Dichotomous Response Strategy

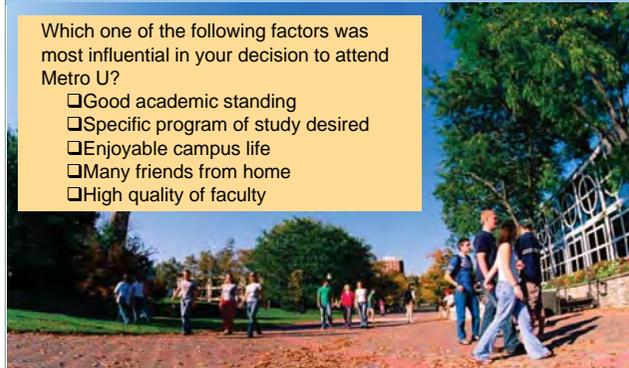


Did you attend the "A Day at College" program at Metro U?

- Yes  
 No

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## Multiple Choice Response Strategy

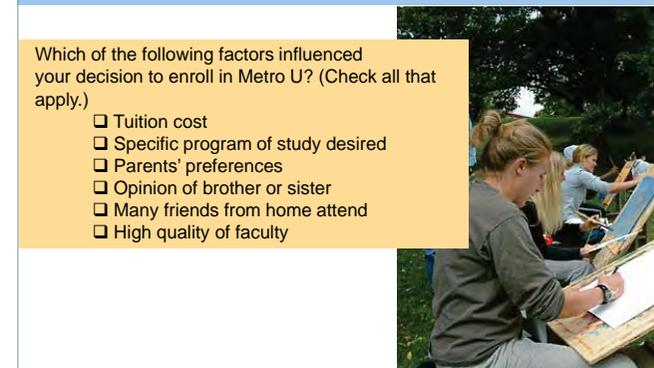


Which one of the following factors was most influential in your decision to attend Metro U?

- Good academic standing  
 Specific program of study desired  
 Enjoyable campus life  
 Many friends from home  
 High quality of faculty

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## Checklist Response Strategy



Which of the following factors influenced your decision to enroll in Metro U? (Check all that apply.)

- Tuition cost  
 Specific program of study desired  
 Parents' preferences  
 Opinion of brother or sister  
 Many friends from home attend  
 High quality of faculty

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## Rating Response Strategy

	Strongly influential	Somewhat influential	Not at all influential
Good academic reputation	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Enjoyable campus life	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Many friends	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
High quality faculty	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Semester calendar	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

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## Ranking



Please rank-order your top three factors from the following list based on their influence in encouraging you to apply to Metro U. Use 1 to indicate the most encouraging factor, 2 the next most encouraging factor, etc.

- \_\_\_\_\_ Opportunity to play collegiate sports  
\_\_\_\_\_ Closeness to home  
\_\_\_\_\_ Enjoyable campus life  
\_\_\_\_\_ Good academic reputation  
\_\_\_\_\_ High quality of faculty

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## Summary of Scale Types

Type	Restrictions	Scale Items	Data Type
<b>Rating Scales</b>			
<b>Simple Category Scale</b>	• Needs mutually exclusive choices	One or more	Nominal
<b>Multiple Choice Single-Response Scale</b>	• Needs mutually exclusive choices • May use exhaustive list or 'other'	Many	Nominal
<b>Multiple Choice Multiple-Response Scale (checklist)</b>	• Needs mutually exclusive choices • Needs exhaustive list or 'other'	Many	Nominal
<b>Likert Scale</b>	• Needs definitive positive or negative statements with which to agree/disagree	One or more	Ordinal
<b>Likert-type Scale</b>	• Needs definitive positive or negative statements with which to agree/disagree	One or more	Ordinal

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## Summary of Scale Types

Type	Restrictions	Scale Items	Data Type
<b>Rating Scales</b>			
<b>Numerical Scale</b>	•Needs concepts with standardized meanings; •Needs number anchors of the scale or end-points •Score is a measurement of graphical space	One or many	Ordinal or Interval
<b>Multiple Rating List Scale</b>	•Needs words that are opposites to anchor the end-points on the verbal scale	Up to 10	Ordinal
<b>Fixed Sum Scale</b>	•Participant needs ability to calculate total to some fixed number, often 100.	Two or more	Interval or Ratio

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## Summary of Scale Types

Type	Restrictions	Scale Items	Data Type
<b>Rating Scales</b>			
<b>Stapel Scale</b>	•Needs verbal labels that are operationally defined or standard.	One or more	Ordinal or Interval
<b>Graphic Rating Scale</b>	•Needs visual images that can be interpreted as positive or negative anchors •Score is a measurement of graphical space from one anchor.	One or more	Ordinal (Interval, or Ratio)

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## Summary of Scale Types

Type	Restrictions	Scale Items	Data Type
<b>Ranking Scales</b>			
<b>Paired Comparison Scale</b>	• Number is controlled by participant's stamina and interest.	Up to 10	Ordinal
<b>Forced Ranking Scale</b>	• Needs mutually exclusive choices.	Up to 10	Ordinal or Interval
<b>Comparative Scale</b>	• Can use verbal or graphical scale.	Up to 10	Ordinal

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## Internet Survey Scale Options

What ONE magazine do you read most often for computing news?  
Please select your answer

- PC Magazine
- Wired
- Computing Magazine
- Computing World
- PC Computing
- Laptop

Multiple Choice, Single Response using pull-down box

Which of the following computing magazines did you look at in the last 30 days?

- PC Magazine
- Wired
- Computing Magazine
- Computing World
- PC Computing
- Laptop

Checklist using checkboxes (may also use radio buttons)

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## Internet Survey Scale Options

Where have you seen advertising for MiniWrite laptop computers?  
Free Response/Open Question using textbox

I plan to purchase a MiniWrite laptop in the next 3 months:  
 Yes  
 No  
Dichotomous Question using radio buttons (may also use pull-down box)

My next laptop computer will have:  
 More memory.  
 More processing speed.  
Paired Comparison using radio buttons (may also use pull-down box)

Which of the following computing magazines did you look at in the last 30 days?  
 PC Magazine  
 Wired  
 Computing Magazine  
 Computing World  
 PC Computing  
 Laptop  
Multiple Choice, Single Response using radio buttons (may also use pull-down box or checkboxes)

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## Internet Survey Scale Options

Please indicate the importance of each of the characteristics in choosing your next laptop. (Select one answer in each row. Scroll to see the complete list of options.)

	Very Important	Neutral (neither important nor unimportant)	Not at all Important
Fast reliable repair service	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Service at my location	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Maintenance by the manufacturer	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Knowledgeable technicians	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Notification of upgrades	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Rating Grid (may also use checkboxes) Requires a single response per line. The longer the list, the more likely the participant must scroll.

From the list below, please choose the three most important service options when choosing your next laptop.  
Fast reliable repair service  
Service at my location  
Maintenance by the manufacturer  
Knowledgeable technicians  
Notification of upgrades

Ranking Question using pull-down box (may also use textboxes, in which ranks are entered) [This question asks for a limited ranking of only three of the listed elements.]

168

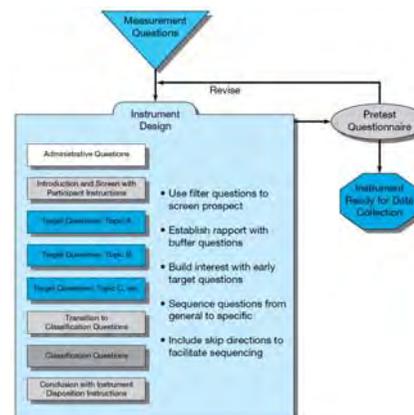
## Sources of Questions

- Handbook of Marketing Scales
- The Gallup Poll Cumulative Index
- Measures of Personality and Social-Psychological Attitudes
- Measures of Political Attitudes

- Index to International Public Opinion
- Sourcebook of Harris National Surveys
- Marketing Scales Handbook
- American Social Attitudes Data Sourcebook

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## Flowchart for Instrument Design: Phase 3



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## Guidelines for Question Sequencing

- Interesting topics early
- Simple topics early
- Sensitive questions later
- Classification questions later
- Transition between topics
- Reference changes limited

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## PicProfile: Branching Question

2. Which of the following attributes do you like about the automobile you just saw? (Select all that apply)

- Overall appeal
- Headroom
- Design
- Color
- Height from the ground
- Other
- None of the above

3. For those items that you selected, how important is each? (Provide one answer for each attribute)

	Extremely important	Important	Neither important nor not important	Not at all important	Don't know
a) Overall appeal	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
b) Height from the ground	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
c) Headroom	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

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## Snapshot: Mobile Questionnaires

10 or fewer questions

Simple question modes

Minimize scrolling

Minimize non-essential content

Minimize distraction



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## Research Thought Leader

“Research that asks consumers what they did and why is incredibly helpful. Research that asks consumers what they are going to do can often be taken with a grain of salt.”

**Al Ries**  
author, co-founder, and chairman  
Ries & Ries.

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## Chapter 14

### SAMPLING



McGraw-Hill/Irwin

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## Learning Objectives

### Understand . . .

- The two premises on which sampling theory is based.
- The accuracy and precision for measuring sample validity.
- The five questions that must be answered to develop a sampling plan.

## Learning Objectives

### Understand . . .

- The two categories of sampling techniques and the variety of sampling techniques within each category.
- The various sampling techniques and when each is used.

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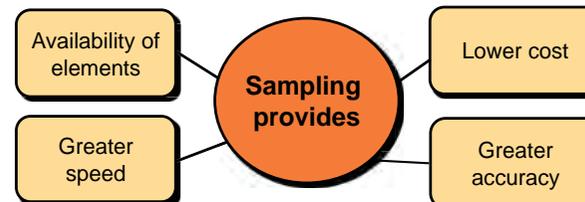
## The Nature of Sampling

- Population
- Population Element
- Sampling Frame
- Census
- Sample

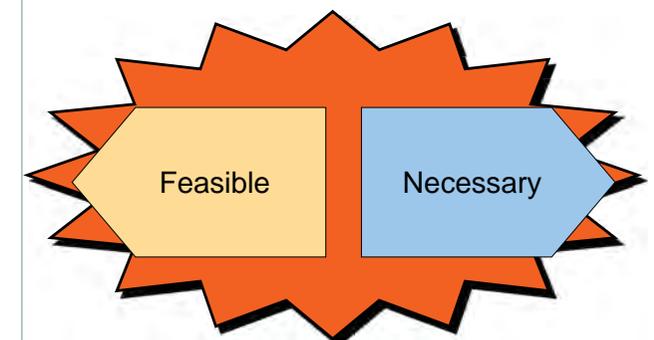


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## Why Sample?



## When Is a Census Appropriate?



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## What Is a Valid Sample?



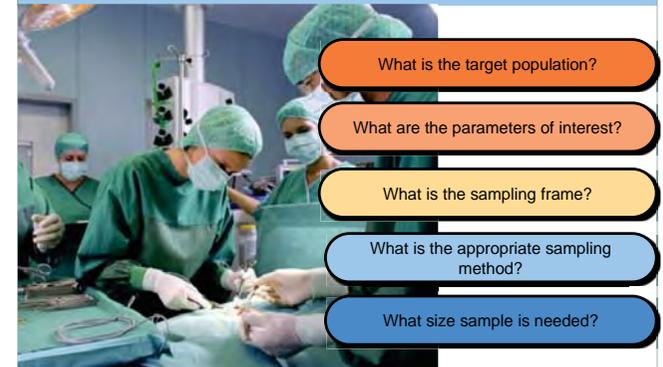
181

## Types of Sampling Designs

Element Selection	Probability	Nonprobability
• Unrestricted	• Simple random	• Convenience
• Restricted	• Complex random	• Purposive
	• Systematic	• Judgment
	• Cluster	• Quota
	• Stratified	• Snowball
	• Double	

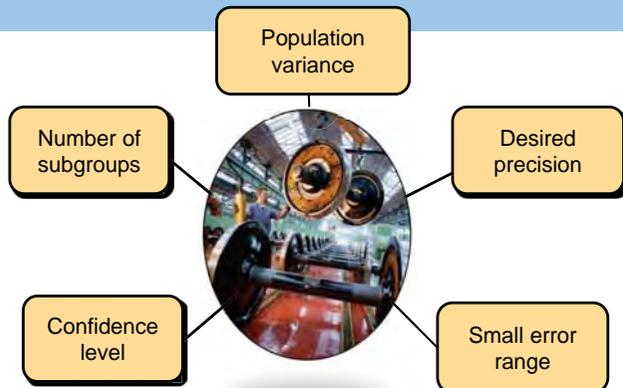
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## Steps in Sampling Design



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## When to Use Larger Sample?



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## Simple Random

### Advantages

- Easy to implement with random dialing

### Disadvantages

- Requires list of population elements
- Time consuming
- Larger sample needed
- Produces larger errors
- High cost

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## Systematic

### Advantages

- Simple to design
- Easier than simple random
- Easy to determine sampling distribution of mean or proportion

### Disadvantages

- Periodicity within population may skew sample and results
- Trends in list may bias results
- Moderate cost

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## Stratified

### Advantages

- Control of sample size in strata
- Increased statistical efficiency
- Provides data to represent and analyze subgroups
- Enables use of different methods in strata

### Disadvantages

- Increased error if subgroups are selected at different rates
- Especially expensive if strata on population must be created
- High cost

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## Cluster

### Advantages

- Provides an unbiased estimate of population parameters if properly done
- Economically more efficient than simple random
- Lowest cost per sample
- Easy to do without list

### Disadvantages

- Often lower statistical efficiency due to subgroups being homogeneous rather than heterogeneous
- Moderate cost

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## Stratified and Cluster Sampling

### Stratified

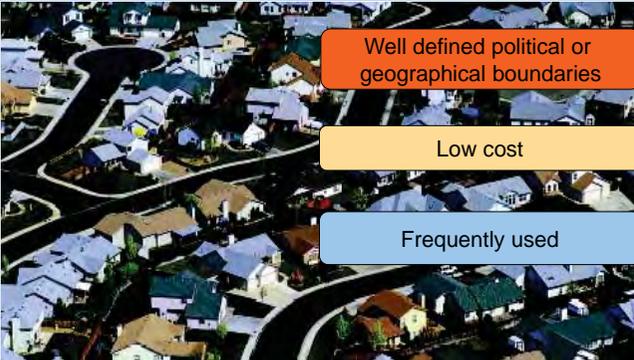
- Population divided into few subgroups
- Homogeneity within subgroups
- Heterogeneity between subgroups
- Choice of elements from within each subgroup

### Cluster

- Population divided into many subgroups
- Heterogeneity within subgroups
- Homogeneity between subgroups
- Random choice of subgroups

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## Area Sampling



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## Double Sampling

### Advantages

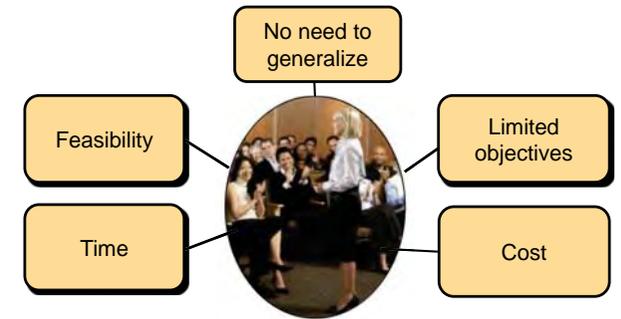
- May reduce costs if first stage results in enough data to stratify or cluster the population

### Disadvantages

- Increased costs if discriminately used

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## Nonprobability Samples



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## Nonprobability Sampling Methods



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## Appendix 15a

### Describing Data Statistically



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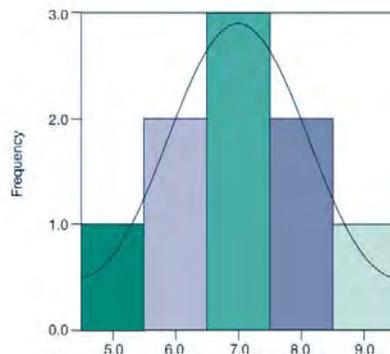
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## Frequencies

	Unit Sales Increase (%)	Frequency	Percentage	Cumulative Percentage
	5	1	11.1	11.1
	6	2	22.2	33.3
	7	3	33.3	66.7
	8	2	22.2	88.9
B	Unit Sales Increase (%)	Frequency	Percentage	Cumulative Percentage
Origin, foreign (1)	6	1	11.1	11.1
	7	2	22.2	33.3
	8	2	22.2	55.5
Origin, foreign (2)	5	1	11.1	66.6
	6	1	11.1	77.7
	7	1	11.1	88.8
	9	1	11.1	100.0
	<b>Total</b>	<b>9</b>	<b>100.0</b>	

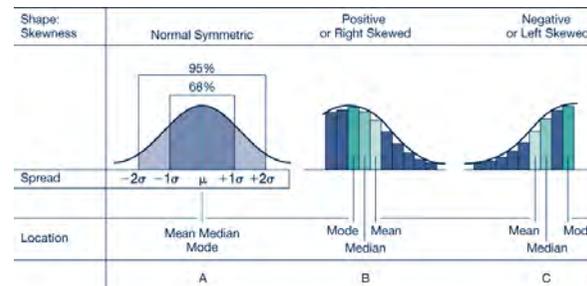
App 15a-195

## Distributions



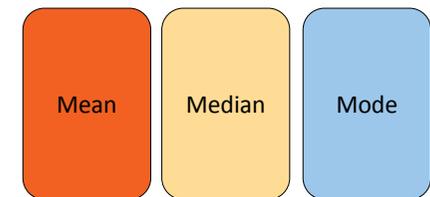
App 15a-196

## Characteristics of Distributions



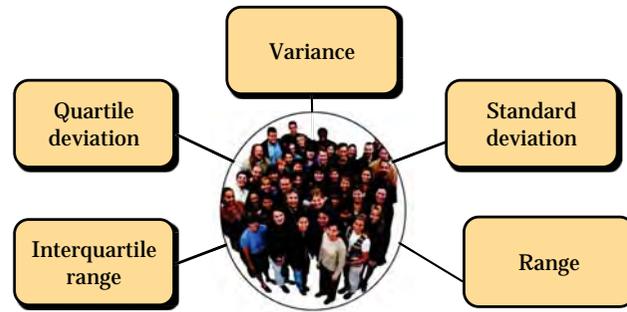
App 15a-197

## Measures of Central Tendency



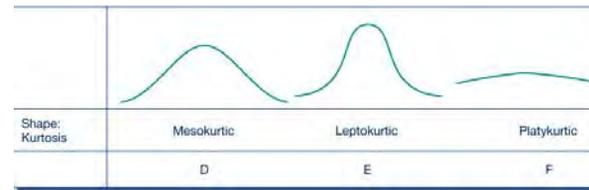
App 15a-198

## Measures of Variability



App 14a-199

## Summarizing Distribution Shape



App 14a-200

## Symbols

Variable	Population	Sample
Mean	$\mu$	$\bar{X}$
Proportion	$\Pi$	$p$
Variance	$\sigma^2$	$s^2$
Standard deviation	$\sigma$	$s$
Size	$N$	$n$
Standard error of the mean	$\sigma_{\bar{X}}$	$S_{\bar{X}}$
Standard error of the proportion	$\sigma_p$	$S_p$

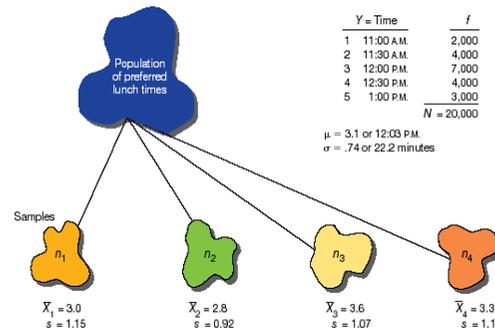
App 14a-201

## Appendix 14a Determining Sample Size



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## Random Samples



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## Increasing Precision

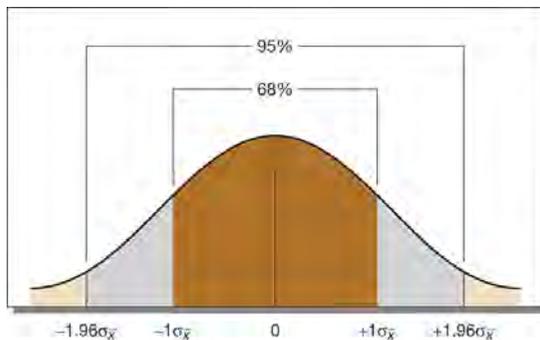
	Reducing the Standard Deviation by 50%	Quadrupling the Sample
$\sigma_{\bar{X}} = \frac{s}{\sqrt{n}}$	$\sigma_{\bar{X}} = \frac{0.74}{\sqrt{10}} = 0.234$	$\sigma_{\bar{X}} = \frac{0.8}{\sqrt{25}} = 0.16$
	$\sigma_{\bar{X}} = \frac{0.37}{\sqrt{10}} = 0.117$	$\sigma_{\bar{X}} = \frac{0.8}{\sqrt{100}} = 0.08$

where  
 $\sigma_{\bar{X}}$  = standard error of the mean  
 $\sigma$  = standard deviation of the sample  
 $n$  = sample size

Note: A 400 percent increase in sample size (from 25 to 100) would yield only a 200 percent increase in precision (from 0.16 to 0.08). Researchers are often asked to increase precision, but the question should be, at what cost? Each of those additional sample elements adds both time and cost to the study.

App 14a-204

## Confidence Levels & the Normal Curve



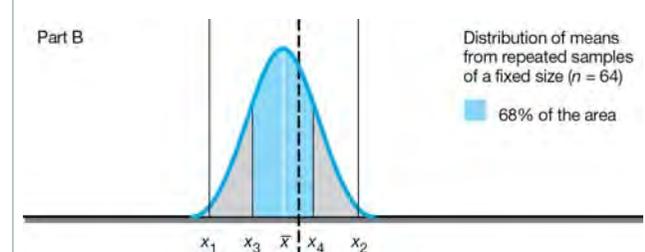
App 14a-205

## Standard Errors

Standard Error (Z score)	% of Area	Approximate Degree of Confidence
1.00	68.27	68%
1.65	90.10	90%
1.96	95.00	95%
3.00	99.73	99%

App 14a-206

## Central Limit Theorem



App 14a-207

## Estimates of Dining Visits

Confidence	Z score	% of Area	Interval Range (visits per month)
68%	1.00	68.27	9.48-10.52
90%	1.65	90.10	9.14-10.86
95%	1.96	95.00	8.98-11.02
99%	3.00	99.73	8.44-11.56

App 14a-208

## Calculating Sample Size for Questions Involving Means

- Precision
- Confidence level
- Size of interval estimate
- Population Dispersion
- Need for FPA

App 14a-209

## Metro U Sample Size for Means

Steps	Information
Desired confidence level	95% ( $z = 1.96$ )
Size of the interval estimate	$\pm .5$ meals per month
Expected range in population	0 to 30 meals
Sample mean	10
Standard deviation	4.1
Need for finite population adjustment	No
Standard error of the mean	$.5/1.96 = .255$
Sample size	$(4.1)^2 / (.255)^2 = 259$

App 14a-210

## Proxies of the Population Dispersion

Previous Research

Pilot or Pretest

Rule-of-thumb: 1/6 of range



App 14a-211

## Metro U Sample Size for Proportions

Steps	Information
Desired confidence level	95% ( $z = 1.96$ )
Size of the interval estimate	$\pm .10$ (10%)
Expected range in population	0 to 100%
Sample proportion with given attribute	30%
Sample dispersion	$Pq = .30(1-.30) = .21$
Finite population adjustment	No
Standard error of the proportion	$.10/1.96 = .051$
Sample size	$.21 / (.051)^2 = 81$

App 14a-212

## Chapter 17

HYPOTHESIS TESTING



McGraw-Hill/Irwin

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## Learning Objectives

**Understand . . .**

- The nature and logic of hypothesis testing.
- A statistically significant difference
- The six-step hypothesis testing procedure.

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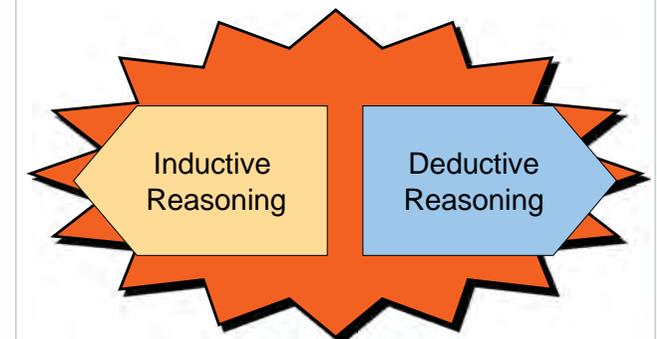
## Learning Objectives

**Understand . . .**

- The differences between parametric and nonparametric tests and when to use each.
- The factors that influence the selection of an appropriate test of statistical significance.
- How to interpret the various test statistics

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## Hypothesis Testing



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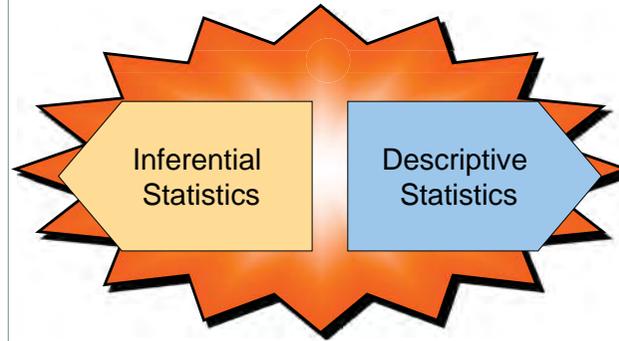
## Hypothesis Testing Finds Truth

“One finds the truth by making a hypothesis and comparing the truth to the hypothesis.”

**David Douglass**  
physicist  
University of Rochester

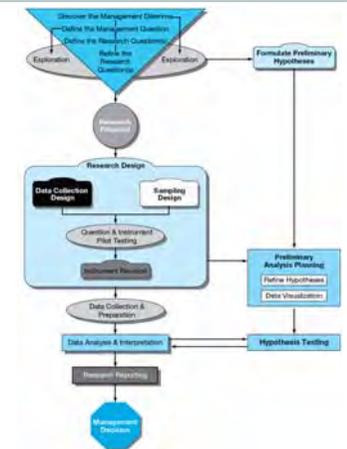
217

## Statistical Procedures



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## Hypothesis Testing and the Research Process



219

## When Data Present a Clear Picture

As Abacus states in this ad, when researchers 'sift through the chaos' and 'find what matters' they experience the 'ah ha!' moment.



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## Approaches to Hypothesis Testing

### Classical statistics

- Objective view of probability
- Established hypothesis is rejected or fails to be rejected
- Analysis based on sample data

### Bayesian statistics

- Extension of classical approach
- Analysis based on sample data
- Also considers established subjective probability estimates

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## Statistical Significance



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## Types of Hypotheses

### Null

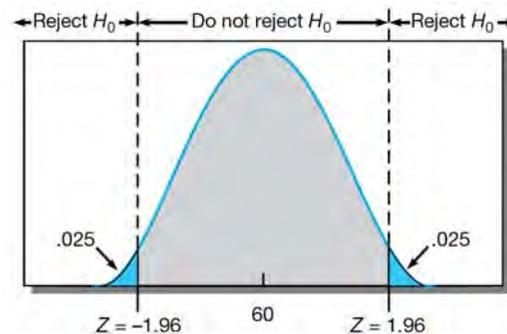
- $H_0: \mu = 50$  mpg
- $H_0: \mu \leq 50$  mpg
- $H_0: \mu \geq 50$  mpg

### Alternate

- $H_A: \mu \neq 50$  mpg
- $H_A: \mu > 50$  mpg
- $H_A: \mu < 50$  mpg

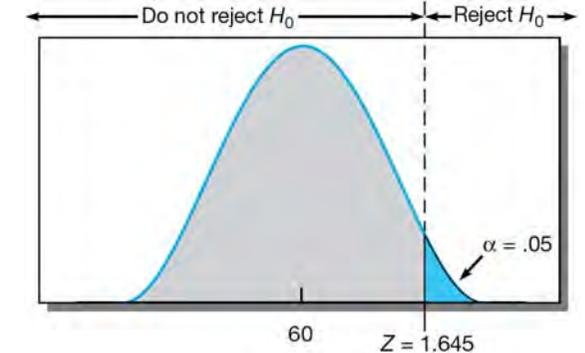
223

## Two-Tailed Test of Significance



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## One-Tailed Test of Significance



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## Decision Rule

Take no corrective action if the analysis shows that one **cannot reject** the null hypothesis.

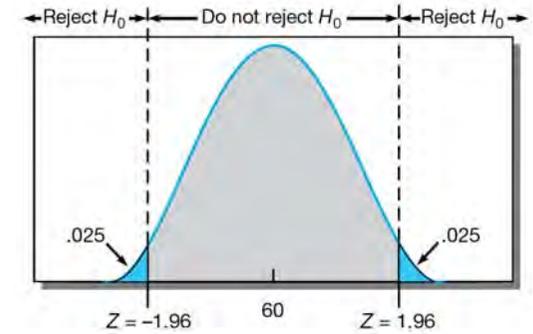
226

## Statistical Decisions

		State of Nature	
		$H_0$ is true	$H_A$ is true
Decision: Accept	$H_0$	Correct decision Power of test Probability = $1 - \alpha$	Type II error Power of test Probability = $\beta$
	$H_A$	Type I error Significance level Probability = $\alpha$	Correct decision Power of test Probability = $1 - \beta$
		Innocent of crime Found not guilty	Guilty of crime Unjustly acquitted
		Innocent Unjustly convicted	Guilty Justly convicted

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## Probability of Making a Type I Error



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## Critical Values

$Z = 1.96$  (significance level = .05)  
 $\bar{X}_c$  = the critical value of the sample mean  
 $\mu$  = the population value stated in  $H_0 = 50$   
 $\sigma_{\bar{X}}$  = the standard error of a distribution of means of samples of 25

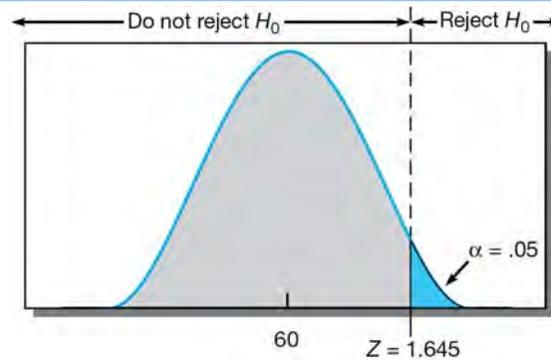
$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}$$

$$-1.96 = \frac{\bar{X}_c - 50}{2} \qquad 1.96 = \frac{\bar{X}_c - 50}{2}$$

$$\bar{X}_c = 46.08 \qquad \bar{X}_c = 53.92$$

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## Probability of Making A Type I Error



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## Factors Affecting Probability of Committing a $\beta$ Error

True value of parameter

Alpha level selected

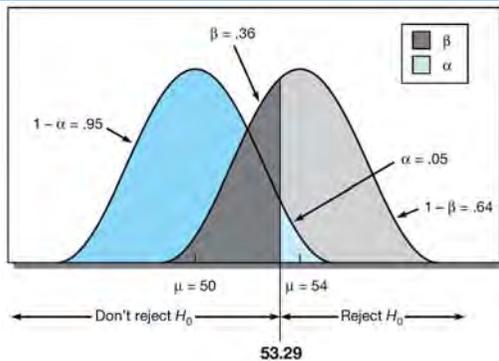
One or two-tailed test used

Sample standard deviation

Sample size

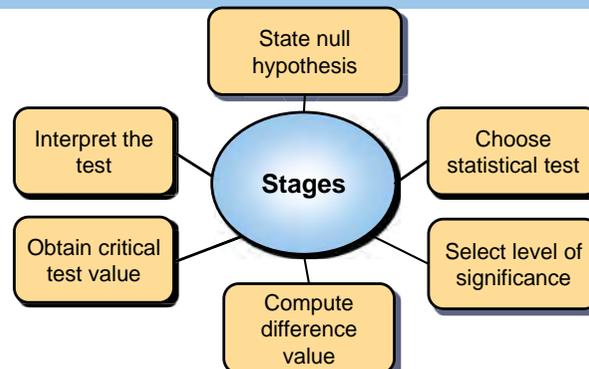
231

## Probability of Making A Type II Error



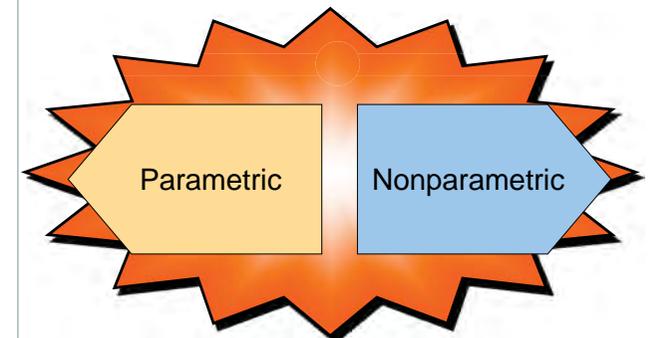
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## Statistical Testing Procedures



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## Tests of Significance



234

## Assumptions for Using Parametric Tests

Independent observations

Normal distribution

Equal variances

Interval or ratio scales

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## Advantages of Nonparametric Tests

Easy to understand and use

Usable with nominal data

Appropriate for ordinal data

Appropriate for non-normal population distributions

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## How to Select a Test

How many samples are involved?

If two or more samples:  
are the individual cases independent or related?

Is the measurement scale  
nominal, ordinal, interval, or ratio?

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## Recommended Statistical Techniques

Measurement Scale	One-Sample Case	Two-Sample Tests		k-Sample Tests	
		Related Samples	Independent Samples	Related Samples	Independent Samples
Nominal	<ul style="list-style-type: none"> <li>Binomial</li> <li><math>\chi^2</math> one-sample test</li> </ul>	<ul style="list-style-type: none"> <li>McNemar</li> </ul>	<ul style="list-style-type: none"> <li>Fisher exact test</li> <li><math>\chi^2</math> two-samples test</li> </ul>	<ul style="list-style-type: none"> <li>Cochran Q</li> </ul>	<ul style="list-style-type: none"> <li><math>\chi^2</math> for k samples</li> </ul>
Ordinal	<ul style="list-style-type: none"> <li>Kolmogorov-Smirnov one-sample test</li> <li>Runs test</li> </ul>	<ul style="list-style-type: none"> <li>Sign test</li> <li>Wilcoxon matched-pairs test</li> </ul>	<ul style="list-style-type: none"> <li>Median test</li> <li>Mann-Whitney U</li> <li>Kolmogorov-Smirnov</li> <li>Wald-Wolfowitz</li> </ul>	<ul style="list-style-type: none"> <li>Friedman two-way ANOVA</li> </ul>	<ul style="list-style-type: none"> <li>Median extension</li> <li>Kruskal-Wallis one-way ANOVA</li> </ul>
Interval and Ratio	<ul style="list-style-type: none"> <li>t-test</li> <li>Z test</li> </ul>	<ul style="list-style-type: none"> <li>t-test for paired samples</li> </ul>	<ul style="list-style-type: none"> <li>t-test</li> <li>Z test</li> </ul>	<ul style="list-style-type: none"> <li>Repeated-measures ANOVA</li> </ul>	<ul style="list-style-type: none"> <li>One-way ANOVA</li> <li>n-way ANOVA</li> </ul>

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## Questions Answered by One-Sample Tests

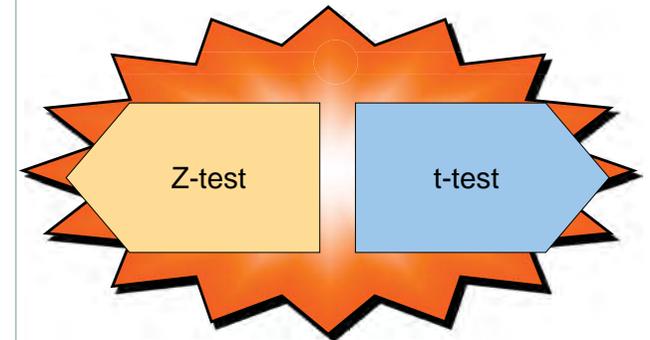
Is there a difference between observed frequencies and the frequencies we would expect?

Is there a difference between observed and expected proportions?

Is there a significant difference between some measures of central tendency and the population parameter?

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## Parametric Tests



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## One-Sample t-Test Example

Null	$H_0: = 50$ mpg
Statistical test	t-test
Significance level	.05, $n=100$
Calculated value	1.786
Critical test value	1.66 (from Appendix C, Exhibit C-2)

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## One Sample Chi-Square Test Example

Living Arrangement	Intend to Join	Number Interviewed	Percent (no. interviewed/200)	Expected Frequencies (percent x 60)
Dorm/fraternity	16	90	45	27
Apartment/rooming house, nearby	13	40	20	12
Apartment/rooming house, distant	16	40	20	12
Live at home	15	30	15	9
Total	60	200	100	60

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## Two-Sample Parametric Tests

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)0}{\sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

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## Two-Sample t-Test Example

	A Group	B Group
Average hourly sales	$X_1 = \$1,500$	$X_2 = \$1,300$
Standard deviation	$s_1 = 225$	$s_2 = 251$

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## Two-Sample t-Test Example

Null	$H_0: A \text{ sales} = B \text{ sales}$
Statistical test	t-test
Significance level	.05 (one-tailed)
Calculated value	1.97, d.f. = 20
Critical test value	1.725 (from Appendix C, Exhibit C-2)

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## Two-Sample Nonparametric Tests: Chi-Square

	Cell Designation Count Expected Values	On-the-Job-Accident		Row Total
		Yes	No	
Smoker	Heavy Smoker	11 12, 8.24	12 4 7.75	16
	Moderate	21 9 7.73	22 6 7.27	15
Nonsmoker		31 13 18.03	32 22 16.97	35
	Column Total	34	32	66

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## Two-Sample Chi-Square Example

Null	There is no difference in distribution channel for age categories.
Statistical test	Chi-square
Significance level	.05
Calculated value	6.86, d.f. = 2
Critical test value	5.99 (from Appendix C, Exhibit C-3)

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## SPSS Cross-Tabulation Procedure

INCOME BY POSSESSION OF MBA				
INCOME	Count	MBA		Row Total
		Yes	No	
High	30	30	60	60.0
Low	10	30	40	40.0
Column Total	40.0	60.0	100.0	

Chi-Square	Value	D.F.	Significance
Pearson	6.25000	1	.01242
Continuity Correction	5.25174	1	.02142
Likelihood Ratio	6.43786	1	.01117
Fleiss-Haenszel	6.18750	1	.02287

Minimum Expected Frequency: 16.000

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## k-Independent-Samples Tests: ANOVA

Tests the null hypothesis that the means of three or more populations are equal.

One-way: Uses a single-factor, fixed-effects model to compare the effects of a treatment or factor on a continuous dependent variable.

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## ANOVA Example

Model Summary					
Source	d.f.	Sum of Squares	Mean Square	F Value	p Value
Model (airline)	2	11644.033	5822.017	28.304	0.0001
Residual (error)	57	11724.550	205.694		
Total	59	23368.583			

Means Table				
	Count	Mean	Std. Dev.	Std. Error
Lufthansa	20	38.950	14.006	3.132
Malaysia Airlines	20	58.900	15.089	3.374
Cathay Pacific	20	72.900	13.902	3.108

All data are hypothetical

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## ANOVA Example Continued

Null	$\mu A1 = \mu A2 = \mu A3$
Statistical test	ANOVA and F ratio
Significance level	.05
Calculated value	28.304, d.f. = 2, 57
Critical test value	3.16 (from Appendix C, Exhibit C-9)

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# CHAPTER 1

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## Introduction

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There are several types of professional economists working in the world today. Academic economists in universities often derive and test theoretical models of various aspects of the economy. Economists in the civil service often study the merits and demerits of policies under consideration by government. Economists employed by a central bank often give advice on whether or not interest rates should be raised, while in the private sector, economists often predict future variables such as exchange rate movements and their effect on company exports.

For all of these economists, the ability to work with data is an important skill. To decide between competing theories, to predict the effect of policy changes, or to forecast what may happen in the future, it is necessary to appeal to facts. In economics, we are fortunate in having at our disposal an enormous amount of facts (in the form of “data”) that we can analyze in various ways to shed light on many economic issues.

The purpose of this book is to present the basics of data analysis in a simple, non-mathematical way, emphasizing graphical and verbal intuition. It focusses on the tools that economists apply in practice (primarily regression) and develops computer skills that are necessary in virtually any career path that the economics student may choose to follow.

To explain further what this book does, it is perhaps useful to begin by discussing what it does **not** do. **Econometrics** is the name given to the study of quantitative tools for analyzing economic data. The field of econometrics is based on probability and statistical theory; it is a fairly mathematical field. This book does not attempt to teach probability and statistical theory. Neither does it contain much mathematical content. In both these respects, it represents a clear departure from traditional econometrics textbooks. Yet, it aims to teach most of the practical tools used by applied econometricians today.

Books that merely teach the student which buttons to press on a computer without providing an understanding of what the computer is doing, are commonly referred to as “cookbooks”. The present book is **not** a cookbook. Some econometricians may interject at this point: “But how can a book teach the student to use the tools of econometrics, without teaching the basics of probability and statistics?” My answer is that much of what the econometrician does in practice can be understood intuitively, without resorting to probability and statistical theory. Indeed, it is a contention of this book that most of the tools econometricians use can be mastered simply through a thorough understanding of the concept of correlation, and its generalization, regression. If a student understands correlation and regression well, then he/she can understand most of what econometricians do. In the vast majority of cases, it can be argued that regression will reveal most of the information in a data set. Furthermore, correlation and regression are fairly simple concepts that can be understood through verbal intuition or graphical methods. They provide the basis of explanation for more difficult concepts, and can be used to analyze many types of economic data.

This book focusses on the **analysis** of economic data; **it is not a book about collecting economic data**. With some exceptions, it treats the data as given, and does not explain how the data is collected or constructed. For instance, it does not explain how national accounts are created or how labor surveys are designed. It simply teaches the reader to make sense out of the data that has been gathered.

Statistical theory usually proceeds from the formal definition of general concepts, followed by a discussion of how these concepts are relevant to particular examples. The present book attempts to do the opposite. That is, **it attempts to motivate general concepts through particular examples**. In some cases formal definitions are not even provided. For instance, P-values and confidence intervals are important statistical concepts, providing measures relating to the accuracy of a fitted regression line (see Chapter 5). The chapter uses examples, graphs and verbal intuition to demonstrate how they might be used in practice. But no formal definition of a P-value nor derivation of a confidence interval is ever given. This would require the introduction of probability and statistical theory, which is not necessary for using these techniques sensibly in practice. For the reader wishing to learn more about the statistical theory underlying the techniques, many books are available; for instance *Introductory Statistics for Business and Economics* by Thomas Wonnacott and Ronald Wonnacott (Fourth edition, John Wiley & Sons, 1990). For those interested in how statistical theory is applied in econometric modeling, *Undergraduate Econometrics* by R. Carter Hill, William E. Griffiths and George G. Judge (Second edition, John Wiley & Sons, 2000) provides a useful introduction.

This book reflects my belief that the use of concrete examples is the best way to teach data analysis. Appropriately, each chapter presents several examples as a means of illustrating key concepts. One risk with such a strategy is that some students might

interpret the presence of so many examples to mean that myriad concepts must be mastered before they can ever hope to become adept at the practice of econometrics. This is not the case. At the heart of this book are only a few basic concepts, and they appear repeatedly in a variety of different problems and data sets. The best approach for teaching introductory econometrics, in other words, is to illustrate its specific concepts over and over again in a variety of contexts.

## Organization of the book

In organizing the book, I have attempted to adhere to the general philosophy outlined above. Each chapter covers a topic and includes a general discussion. However, most of the chapter is devoted to empirical examples that illustrate and, in some cases, introduce important concepts. Exercises, which further illustrate these concepts, are included in the text. Data required to work through the empirical examples and exercises can be found in the website which accompanies this book <http://www.wileyurope.com/go/koopdata2ed>. By including real-world data, it is hoped that students will not only replicate the examples, but will feel comfortable extending and/or experimenting with the data in a variety of ways. Exposure to real-world data sets is essential if students are to master the conceptual material and apply the techniques covered in this book.

The empirical examples in this book are designed for use in conjunction with the computer package Excel. The website associated with this book contains Excel files. Excel is a simple and common software package. It is also one that students are likely to use in their economic careers. However, the data can be analyzed using many other computer software packages, not just Excel. Many of these packages recognize Excel files and the data sets can be imported directly into them. Alternatively, the website also contains all of the data files in ASCII text form. Appendix B at the end of the book provides more detail about the data.

Mathematical material has been kept to a minimum throughout this book. In some cases, a little bit of mathematics will provide additional intuition. For students familiar with mathematical techniques, appendices have been included at the end of some chapters. However, students can choose to omit this material without any detriment to their understanding of the basic concepts.

The content of the book breaks logically into two parts. Chapters 1–7 cover all the basic material relating to graphing, correlation and regression. A very short course would cover only this material. Chapters 8–12 emphasize time series topics and analyze some of the more sophisticated econometric models in use today. The focus on the underlying intuition behind regression means that this material should be easily accessible to students. Nevertheless, students will likely find that these latter chapters are more difficult than Chapters 1–7.

## Useful background

As mentioned, this book assumes very little mathematical background beyond the pre-university level. Of particular relevance are:

1. Knowledge of simple equations. For instance, the equation of a straight line is used repeatedly in this book.
2. Knowledge of simple graphical techniques. For instance, this book is full of graphs that plot one variable against another (i.e. standard  $XY$ -graphs).
3. Familiarity with the summation operator is useful occasionally.
4. In a few cases, logarithms are used.

For the reader unfamiliar with these topics, the appendix at the end of this chapter provides a short introduction. In addition, these topics are discussed elsewhere, in many introductory mathematical textbooks.

This book also has a large computer component, and much of the computer material is explained in the text. There are myriad computer packages that could be used to implement the procedures described in this book. In the places where I talk directly about computer programs, I will use the language of spreadsheets and, particularly, that most common of spreadsheets, Excel. I do this largely because the average student is more likely to have knowledge of and access to a spreadsheet rather than a specialized statistics or econometrics package such as E-views, Stata or MicroFit.<sup>1</sup> I assume that the student knows the basics of Excel (or whatever computer software package he/she is using). In other words, students should understand the basics of spreadsheet terminology, be able to open data sets, cut, copy and paste data, etc. If this material is unfamiliar to the student, simple instructions can be found in Excel's on-line documentation. For computer novices (and those who simply want to learn more about the computing side of data analysis) *Computing Skills for Economists* by Guy Judge (John Wiley & Sons, 2000) is an excellent place to start.

## Appendix 1.1: Mathematical concepts used in this book

This book uses very little mathematics, relying instead on intuition and graphs to develop an understanding of key concepts (including understanding how to interpret the numbers produced by computer programs such as Excel). For most students, previous study of mathematics at the pre-university level should give you all the background knowledge you need. However, here is a list of the concepts used in this book along with a brief description of each.

## The equation of a straight line

Economists are often interested in the relationship between two (or more) variables. Examples of variables include house prices, gross domestic product (GDP), interest rates, etc. In our context a variable is something the economist is interested in and can collect data on. I use capital letters (e.g.  $Y$  or  $X$ ) to denote variables. A very general way of denoting a relationship is through the concept of a function. A common mathematical notation for a function of  $X$  is  $f(X)$ . So, for instance, if the economist is interested in the factors that explain why some houses are worth more than others, he/she may think that the price of a house depends on the size of the house. In mathematical terms, he/she would then let  $Y$  denote the variable “price of the house” and  $X$  denote the variable “size of the house” and the fact that  $Y$  depends on  $X$  is written using the notation:

$$Y = f(X)$$

This notation should be read “ $Y$  is a function of  $X$ ” and captures the idea that the value for  $Y$  depends on the value of  $X$ . There are many functions that one could use, but in this book I will usually focus on linear functions. Hence, I will not use this general “ $f(X)$ ” notation in this book.

The equation of a straight line (what was called a “linear function” above) is used throughout this book. Any straight line can be written in terms of an equation:

$$Y = \alpha + \beta X$$

where  $\alpha$  and  $\beta$  are **coefficients**, which determine a particular line. So, for instance, setting  $\alpha = 1$  and  $\beta = 2$  defines one particular line while  $\alpha = 4$  and  $\beta = -5$  defines a different line.

It is probably easiest to understand straight lines by using a graph (and it might be worthwhile for you to sketch one at this stage). In terms of an  $XY$  graph (i.e. one which measures  $Y$  on the vertical axis and  $X$  on the horizontal axis) any line can be defined by its intercept and slope. In terms of the equation of a straight line,  $\alpha$  is the intercept and  $\beta$  the slope. The intercept is the value of  $Y$  when  $X = 0$  (i.e. point at which the line cuts the  $Y$ -axis). The slope is a measure of how much  $Y$  changes when  $X$  is changed. Formally, it is the amount  $Y$  changes when  $X$  changes by one unit. For the student with a knowledge of calculus, the slope is the first derivative,  $\frac{dY}{dX}$ .

## Summation notation

At several points in this book, subscripts are used to denote different observations from a variable. For instance, a labor economist might be interested in the wage of every one of 100 people in a certain industry. If the economist uses  $Y$  to denote this variable, then he/she will have a value of  $Y$  for the first individual, a value of  $Y$  for the second individual, etc. A compact notation for this is to use subscripts so that  $Y_1$

is the wage of the first individual,  $Y_2$  the wage of the second individual, etc. In some contexts, it is useful to speak of a generic individual and refer to this individual as the  $i$ -th. We can then write,  $Y_i$  for  $i = 1, \dots, 100$  to denote the set of wages for all individuals.

With the subscript notation established, summation notation can now be introduced. In many cases we want to add up observations (e.g. when calculating an average you add up all the observations and divide by the number of observations). The Greek symbol,  $\Sigma$ , is the summation (or “adding up”) operator and superscripts and subscripts on  $\Sigma$  indicate the observations that are being added up. So, for instance,

$$\sum_{i=1}^{100} Y_i = Y_1 + Y_2 + \dots + Y_{100}$$

adds up the wages for all of the 100 individuals. As other examples,

$$\sum_{i=1}^3 Y_i$$

adds up the wages for the first 3 individuals and

$$\sum_{i=47}^{48} Y_i$$

adds up the wages for the 47th and 48th individuals.

Sometimes, where it is obvious from the context (usually when summing over all individuals), the subscript and superscript will be dropped and I will simply write:

$$\sum Y_i.$$

## Logarithms

For various reasons (which are explained later on), in some cases the researcher does not work directly with a variable but with a transformed version of this variable. Many such transformations are straightforward. For instance, in comparing the incomes of different countries the variable GDP per capita is used. This is a transformed version of the variable GDP. It is obtained by dividing GDP by population.

One particularly common transformation is the logarithmic one. The logarithm (to the base  $B$ ) of a number,  $\mathcal{A}$ , is the power to which  $B$  must be raised to give  $\mathcal{A}$ . The notation for this is:  $\log_B(\mathcal{A})$ . So, for instance, if  $B = 10$  and  $\mathcal{A} = 100$  then the logarithm is 2 and we write  $\log_{10}(100) = 2$ . This follows since  $10^2 = 100$ . In economics, it is common to work with the so-called natural logarithm which has  $B = e$  where  $e \approx 2.71828$ . We will not explain where  $e$  comes from or why this rather unusual-looking base is chosen. The natural logarithm operator is denoted by  $\ln$ ; i.e.  $\ln(\mathcal{A}) = \log_e(\mathcal{A})$ .

In this book, you do not really have to understand the material in the previous paragraph. The key thing to note is that the natural logarithmic operator is a common one (for reasons explained later on) and it is denoted by  $\ln(\mathcal{A})$ . In practice, it can be easily calculated in a spreadsheet such as Excel (or on a calculator).

## Endnote

1. I expect that most readers of this book will have access to Excel (or a similar spreadsheet or statistics software package) through their university computing labs or on their home computers (note, however, that some of the methods in this book require the Excel Analysis ToolPak add-in which is not included in some basic installations of Microsoft Works). However, computer software can be expensive and, for the student who does not have access to Excel and is financially constrained, there is an increasing number of free statistics packages designed using open source software. R. Zelig, which is available at <http://gking.harvard.edu/zelig/>, is a good example of such a package.

# ANALYSIS OF ECONOMIC DATA

by Gary Koop

## Chapter 3: Correlation

Correlation measures numerically the relationship between two variables X and Y (e.g. population density and deforestation).

Correlation between X and Y is symbolised by r or  $r_{XY}$ .

$$r = \frac{\sum(Y_i - \bar{Y})(X_i - \bar{X})}{\sqrt{\sum(Y_i - \bar{Y})^2} \sqrt{\sum(X_i - \bar{X})^2}}$$

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## Understanding Correlation

Example: Y = deforestation, X = population density we obtain  $r = .66$ .

### What does the fact r is positive mean?

- There is a positive relationship between population density and deforestation.
- Countries with high population densities also tend to have high deforestation rates.
- Countries with low population densities tend to have low deforestation rates.
- Deforestation and population densities both vary across countries. The high/low variation in deforestation rates tends to match up with the high/low variation in population densities.

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## Properties of Correlation

- r lies between -1 and +1.
- Positive values of r indicate positive correlation between X and Y, negative values indicate negative correlation,  $r = 0$  implies X and Y are uncorrelated.
- Larger positive values of r indicate stronger positive correlation.  $r = 1$  indicates perfect positive correlation.
- More negative values of r indicate stronger negative correlation.  $r = -1$  indicates perfect negative correlation.
- The correlation between Y and X is the same as the correlation between X and Y.
- The correlation between any variable and itself is 1.
- Correlation only measures linear relations

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## Understanding Correlation (cont.)

### What does the magnitude of r mean?

$r^2$  measures the proportion of the variance in deforestation that matches up with (or is explained by) the variance in population density.

$$r^2 = .66^2 = .44.$$

44% of the cross-country variation in deforestation can be explained by the cross-country variation in population density.

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## Why are Variables Correlated?

### Correlation does not necessarily imply causality.

#### Example:

The correlation between workers' education levels and wages is strongly positive.

Does this mean education "causes" higher wages? We can't know for sure.

- **Possibility 1: Education improves skills and more skilled workers get better paying jobs.**

Education causes wages to increase.

- **Possibility 2: Individuals are born with quality A which is relevant for success in education and on the job (e.g. intelligence or talent or determination, etc.).**

Quality A (NOT education) causes wages to increase.

## Why are Variables Correlated? (cont.)

#### Example:

Data on N=546 houses sold in Windsor, Canada

Y = sales price of a house

X = the size of the lot the house is on

$r_{XY} = .54$

- Houses with large lots tend to be worth more than houses with small lots.
- Economic theory tells us that the price of a house should depend on its characteristics.
- Economic theory suggests X is causing Y.
- Here economic theory suggests that correlation does imply causality.

## Why are Variables Correlated? (cont.)

#### Example:

#### Assume:

- Cigarette smoking causes cancer.
- Drinking alcohol does not cause cancer.
- Smokers tend to drink more alcohol.

Suppose we collected data on many elderly people on how much they smoked (X), whether they had cancer (Y) and how much they drank (Z). What correlations would we find?

$r_{XY} > 0$  Direct causality.

$r_{YZ} > 0$  This is does NOT reflect causality.

## Why are Variables Correlated? (cont.)

#### Example:

High rural population density (X) causes farmers to clear new land in forested areas (Z) which in turn causes deforestation (Y).

Here we would find  $r_{XY} > 0$  and  $r_{ZY} > 0$ .

X (population density) is an indirect (or proximate) cause of Y (deforestation).

Z (agricultural clearance) is a direct (or immediate) cause of Y (deforestation).

## Why are Variables Correlated? Summary

- Correlations can be very suggestive, but cannot on their own establish causality.
- Correlation + a sensible theory suggests (but does not prove) causality

## Understanding Correlation through XY-plots

### Figure 3.1: House price versus lot size

Think of drawing a straight line that best fits the points in the XY-plot. It will have positive slope.

Positive slope=positive relationship=positive correlation.

### Figure 3.2: XY plot with $r_{XY} = 1$

All points fit on a straight upward sloping line.

## Understanding Correlation through XY-plots (continued)

### Figure 3.3: XY plot with $r_{XY} = .51$

Points still exhibit an upward sloping pattern, but much more scattered.

### Figure 3.4: XY plot with $r_{XY} = 0$

Completely random scattering of points.

### Figure 3.5: XY plot with $r_{XY} = -.51$

Think of drawing a straight line that best fits the points in the XY-plot. It will have negative slope.

Negative slope=negative relationship=negative correlation.

## Correlation Among Several Variables

Correlation relates precisely two variables.

What to do with three or more? Usually use regression (next chapter).

Or you can calculate the correlation between every possible pair of variables.

Example: Three variables: X, Y and Z.

Can calculate three correlations:  $r_{xy}$ ,  $r_{xz}$  and  $r_{yz}$ .

Four variables: X, Y, Z and W.

Can calculate six correlation:  $r_{xy}$ ,  $r_{xz}$ ,  $r_{xw}$ ,  $r_{yz}$ ,  $r_{yw}$  and  $r_{zw}$ .

M variables:  $M \times (M-1) / 2$  correlations.

## A Correlation Matrix

Example:

Column 1 = X, Column 2 = Y, Column 3 = Z

	<i>Column 1</i>	<i>Column 2</i>	<i>Column 3</i>
<i>Column 1</i>	1		
<i>Column 2</i>	0.3182369	1	
<i>Column 3</i>	-0.1309744	0.0969959	1

$$r_{xy} = 0.3182369$$

$$r_{xz} = -0.1309744$$

$$r_{yz} = 0.0969959$$

# ANALYSIS OF ECONOMIC DATA

by Gary Koop

## Chapter 4: An Introduction to Simple Regression

- Regression is the most common tool of the applied economist.
- Used to help understand the relationships between many variables.
- We begin with simple regression to understand the relationship between two variables, X and Y.

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### Simple Regression: Theory

Assume a linear relationship exists between Y and X:

$$Y = \alpha + \beta X$$

$\alpha$  = intercept of line

$\beta$  = slope of line

Example:

Y = output of a good, X = labour input

$$Y = .8 \times X$$

$\alpha = 0$

$\beta = .8$  = marginal product of labour.

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## Regression as a Best Fitting Line

Example:

See Figure 2.3: XY-plot of deforestation versus population density.

Regression fits a line through the points in the XY-plot that best captures the relationship between deforestation and population density.

Question: What do we mean by “best fitting” line?

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### Simple Regression: Theory (cont.)

1. Even if straight line relationship were true, we would never get all points on an XY-plot lying precisely on it due to measurement error.
2. True relationship probably more complicated, straight line may just be an approximation.
3. Important variables which affect Y may be omitted.

Due to 1., 2. and 3. we add an error.

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## The Simple Regression Model

$$Y = \alpha + \beta X + e$$

where  $e$  is an error.

What we know:  $X$  and  $Y$ .

What we do not know:  $\alpha$ ,  $\beta$  or  $e$ .

Regression analysis uses data ( $X$  and  $Y$ ) to make a guess or estimate of what  $\alpha$  and  $\beta$  are.

Notation:  $\hat{\alpha}$  and  $\hat{\beta}$  are the estimates of  $\alpha$  and  $\beta$ .

## Distinction Between Errors and Residuals

**True Regression Line:**

$$Y = \alpha + \beta X + e$$

$$e = Y - \alpha - \beta X$$

$e = \text{error}$

**Estimated Regression Line:**

$$Y = \hat{\alpha} + \hat{\beta} X + u$$

$$u = Y - \hat{\alpha} - \hat{\beta} X$$

$u = \text{residual}$

## How do we choose $\hat{\alpha}$ and $\hat{\beta}$ ?

Consider the following reasoning:

1. We want to find a best fitting line through the XY-plot.
2. With more than two points it is not possible to find a line that fits perfectly through all points. (See Figure 4.1)
3. Hence, find “best fitting” line which makes the residuals as small as possible.
4. What do we mean by “as small as possible”?  
The one that minimizes the sum of squared residuals.
5. Hence, we obtain the “ordinary least squares” or OLS estimator.

## Derivation of OLS Estimator

- We have data on  $i=1, \dots, N$  individuals which we call  $Y_i$  and  $X_i$ .
- Any line we fit/choice of  $\hat{\alpha}$  and  $\hat{\beta}$  will yield residuals  $u_i$ .
- Sum of squared residuals =  $SSR = \sum u_i^2$ .
- OLS estimator chooses  $\hat{\alpha}$  and  $\hat{\beta}$  to minimise SSR.

Solution:

$$\hat{\beta} = \frac{\sum (Y_i - \bar{Y})(X_i - \bar{X})}{\sum (X_i - \bar{X})^2}$$

and

$$\hat{\alpha} = \bar{Y} - \hat{\beta} \times \bar{X}$$

## Jargon of Regression

- $Y$  = dependent variable.
- $X$  = explanatory (or independent) variable.
- $\alpha$  and  $\beta$  are coefficients.
- $\hat{\alpha}$  and  $\hat{\beta}$  are OLS estimates of coefficients
- “Run a regression of  $Y$  on  $X$ ”

## Regression and Causality

### How to decide which is dependent variable

- Ideally, explanatory variable should be the one which causes/influences the dependent variable.
- Ideally,  $X$  causes  $Y$ .

If you can, build models where causality assumptions make sense

Examples:

- Increases in  $X$  = population density cause  $Y$  = deforestation to increase (not vice versa).
- Increasing  $X$  = the lot size of a house causes  $Y$  = its value to increase (not vice versa).
- Increasing  $X$  = advertising expenditures causes  $Y$  = company sales to increase (not vice versa).

## Regression and Causality (cont.)

In practice, great care must be taken in interpreting regression results as reflecting causality. Why?

- In some cases, your assumption that  $X$  causes  $Y$  may be wrong.
- In some cases, you may not know whether  $X$  causes  $Y$ .
- In some cases,  $X$  may cause  $Y$  but  $Y$  may also cause  $X$  (e.g. exchange rates and interest rates).
- In some cases, the whole concept of causality may be inappropriate.

Formally, the question regression addresses is: “How much of the variability in  $Y$  can be explained by  $X$ ?”

## Interpreting OLS Estimates

$$Y = \hat{\alpha} + \hat{\beta}X + u$$

Ignore  $u$  for the time being (focus on regression line)

### Interpretation of $\hat{\alpha}$

- Estimated value of  $Y$  if  $X = 0$ .
- This is often not of interest.

Example:

$X$  = lot size,  $Y$  = house price

$\hat{\alpha}$  = estimated value of a house with lot size = 0

## Interpreting OLS Estimates (cont.)

$$Y = \hat{\alpha} + \hat{\beta}X + u$$

### Interpretation of $\hat{\beta}$

1.  $\hat{\beta}$  is estimate of the marginal effect of X on Y

2. Using regression model:

$$\frac{dY}{dX} = \hat{\beta}$$

3. A measure of how much Y tends to change when you change X.

4. "If X changes by 1 unit then Y tends to change by  $\hat{\beta}$  units", where "units" refers to what the variables are measured in (e.g. \$, £, %, hectares, metres, etc.).

## Example: Deforestation and Population Density

Development economists have theories which imply that increasing population density should increase deforestation.

Thus:

Y = deforestation (annual % change) = dependent variable

X = population density (people per thousand hectares) = explanatory variable

Using data on N = 70 tropical countries we find:

$$\hat{\beta} = 0.000842$$

## Example: Deforestation and Population Density (continued)

### Interpretation of $\hat{\beta}$

a) "If population density increases by 1 person per 1,000 hectares, then deforestation will tend to increase by .000842% per year"

b) "If population density increases by 100 people per 1,000 hectares, then deforestation will tend to increase by .0842% per year"

Note: if deforestation increased by .0842% per year an additional 5% of the forest will be lost over 50 years.

## Example: Cost of Production in the Electric Utility Industry

Data on N = 123 electric utility companies in the U.S.

Y = cost of production (millions of \$)

X = output (thousands of kilowatt hours, Kwh)

$$\hat{\beta} = .005$$

"Increasing output by 1,000 Kwh tends to increase costs by \$5,000"

Note:  $.005 \times 1,000,000 = 5,000$

"Decreasing output by 1,000 Kwh tends to decrease costs by \$5,000"

## Example: The Effect of Advertising on Sales

Data on N = 84 companies in the U.S.

Y = sales (millions of \$)

X = advertising expenditure (millions of \$)

$$\hat{\beta} = .218$$

“Increases in advertising of \$1,000,000 are associated with increases in sales of \$218,000.”

## Residual Analysis

“How well does the regression line fit through the points on the XY-plot?”

“Are there any outliers which do not fit the general pattern?”

### Concepts:

1. Fitted value of dependent variable:

$$\hat{Y}_i = \hat{\alpha} + \hat{\beta} \times X_i$$

- $Y_i$  does not lie on the regression line,  $\hat{Y}_i$  does lie on line.
- What would you do if you did not know  $Y_i$ , but only  $X_i$  and wanted to predict what  $Y_i$  was? Answer  $\hat{Y}_i$

2. Residual,  $u_i$ , is:  $Y_i - \hat{Y}_i$

## Residual Analysis (cont.)

- Good fitting models have small residuals (i.e. small SSR)
- If residual is big for one observation (relative to other observations) then it is an outlier.
- Looking at fitted values and residuals can be very informative.

## R<sup>2</sup>: A Measure of Fit

Intuition:

“Variability” = (e.g.) how deforestation rates vary across countries

Total variability in dependent variable Y =

Variability explained by the explanatory variable (X) in the regression

+

Variability that cannot be explained and is left as an error.

## R<sup>2</sup>: A Measure of Fit (cont.)

In mathematical terms,

$$TSS = RSS + SSR$$

where TSS = Total sum of squares

$$TSS = \sum (Y_i - \bar{Y})^2$$

Note similarity to formula for variance.

RSS = Regression sum of squares

$$RSS = \sum (\hat{Y}_i - \bar{Y})^2$$

SSR = sum of squared residuals

## Properties of R<sup>2</sup>

- $0 \leq R^2 \leq 1$
- $R^2=1$  means perfect fit. All data points exactly on regression line (i.e. SSR=0).
- $R^2=0$  means X does not have any explanatory power for Y whatsoever (i.e. X has no influence on Y).
- Bigger values of R<sup>2</sup> imply X has more explanatory power for Y.
- R<sup>2</sup> is equal to the correlation between X and Y squared (i.e.  $R^2=r_{xy}^2$ )

## R<sup>2</sup>: A Measure of Fit (cont.)

$$R^2 = 1 - \frac{SSR}{TSS}$$

or (equivalently):

$$R^2 = \frac{RSS}{TSS}$$

R<sup>2</sup> is a measure of fit (i.e. how well does the regression line fit the data points)

## Properties of R<sup>2</sup> (cont.)

R<sup>2</sup> measures the proportion of the variability in Y that can be explained by X.

Example:

In regression of Y = deforestation on X = population density we obtain  $R^2=0.44$

“44% of the cross-country variation in deforestation rates can be explained by the cross-country variation in population density”

## Nonlinearity

So far regression of Y on X:

$$Y = \alpha + \beta X + e$$

Could do regression of Y (or  $\ln(Y)$  or  $Y^2$ ) on  $X^2$  (or  $1/X$  or  $\ln(X)$  or  $X^3$ , etc.) and same techniques and results would hold.

E.g.

$$Y = \alpha + \beta X^2 + e$$

How might you know if relationship is nonlinear?

Answer: Careful examination of XY-plots or residual plots.

Figures 4.2, 4.3 and 4.4.

## Chapter 5: Statistical Aspects of Regression

$\hat{\alpha}$  and  $\hat{\beta}$  are only estimates of  $\alpha$  and  $\beta$

**Key question:** How accurate are these estimates?

Statistical procedures allow us to formally address this question.

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## What Factors Affect Accuracy of OLS Estimates?

Graphical Intuition:

- Figure 5.1 (small number of data points)
- Figure 5.2 (large number of data points but very scattered)
- Figure 5.3 (large number of data points but not very scattered)
- Figure 5.4 (large number of data points, but clustered near one value for X)

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## What Factors Affect Accuracy of OLS Estimates?

Consider fitting a line through the XY-plots in Figures 5.1-5.4.

You would be most confident in the line you fit in Figure 5.3

*Larger number of data points + less scattering (i.e. less variability in errors) + more variability in X = more accurate estimates.*

**Note:** Figures 5.1, 5.2, 5.3 and 5.4 all contain artificially generated data with  $\alpha=0$ ,  $\beta=1$ .

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## A Confidence Interval for $\beta$

- Uncertainty about accuracy of the estimate  $\hat{\beta}$  can be summarised in a “confidence interval”
- 95% confidence interval for  $\beta$  is given by:

$$[\hat{\beta} - t_{\alpha/2} s_b, \hat{\beta} + t_{\alpha/2} s_b]$$

- $t_{\alpha/2}$  is a “critical value” from the “Student t-distribution” --- calculated automatically in Excel
- $s_b$  = standard error of  $\hat{\beta}$  is a measure of the accuracy of  $\hat{\beta}$

$$s_b = \sqrt{\frac{SSR}{(N-2) \times \sum (X_i - \bar{X})^2}}$$

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## A Confidence Interval for $\hat{\beta}$ (cont.)

- $t_b$  controls the confidence level (e.g.  $t_b$  is bigger for 95% confidence than 90%).
- $s_b$  varies directly with SSR (i.e. how variable the residuals are)
- $s_b$  varies inversely with N, the number of data points
- $s_b$  varies inversely with  $\sum(X_i - \bar{X})^2$ , which is related to the variance/variability of X.

Note: Excel calculates the confidence interval for you and labels bounds of confidence interval as “Lower 95%” and “Upper 95%”

## Intuition of Confidence Interval

- Useful (but formally incorrect) intuition: “There is a 95% probability that the true value of  $\beta$  lies in the confidence interval”.
- Correct intuition: “If you repeatedly use the above formula for calculating a confidence interval, 95% of the intervals you construct will contain the true value for  $\beta$ ”.
- Can choose any level of confidence you want (e.g. 90%, 99%).

### Example: Confidence Intervals for the Data sets in Figures 5.1-5.4

Data Set	$\hat{\beta}$	90% Confid. Interval	95% Confid. Interval	99% Confid. Interval
Figure 5.1	.91	[-.92,2.75]	[-1.57,3.39]	[-3.64,5.47]
Figure 5.2	1.04	[.75,1.32]	[.70,1.38]	[.59,1.49]
Figure 5.3	1.00	[.99,1.01]	[.99,1.02]	[.98,1.03]
Figure 5.4	1.52	[-1.33,4.36]	[-1.88,4.91]	[-2.98,6.02]

### Example: The Regression of Deforestation on Population Density

Y = deforestation

X = population density

$$\hat{\beta} = .000842$$

95% Confidence interval: [.00061,.001075]

## Example: The Regression of Lot Size on House Price

OLS results:

$$Y = 34,136 + 6.59X,$$

- The OLS estimate of the marginal effect of X on Y is 6.59.
- “Increasing lot size by an extra square foot is associated with a \$6.59 increase in house price.”
- The 95% confidence interval for  $\beta$  is [5.72,7.47].
- “We are 95% confident that the effect of lot size on house is at least \$5.72 and at most \$7.47.”

## Hypothesis Testing (cont.)

Q: What do we mean by “big” and “small”?

A: Look at P-value.

- If P-value  $\leq .05$  then t is “big” and conclude  $\beta \neq 0$ .
- If P-value  $> .05$  then t is “small” and conclude  $\beta = 0$ .
- Useful (but formally incorrect) intuition: P-value measures the probability that  $\beta = 0$ .
- $.05 = 5\% =$  level of significance
- Other levels of significance (e.g. 1% or 10%) occasionally used

## Hypothesis Testing

- Test whether  $\beta=0$  (i.e. whether X has any explanatory power)
- One way of doing it: look at confidence interval, check whether it contains zero. If no, then you are confident  $\beta \neq 0$ .
- Alternative (equivalent) way is to use “t-statistic” (often called “t-ratio”)

$$t = \frac{\hat{\beta}}{s_b}$$

- “Big” values for t indicate  $\beta \neq 0$ .
- “Small” values for t indicate it  $\beta = 0$ .  
(more concretely:  $\beta$  might be 0)

## Example: The Regression of Deforestation on Population Density (cont.)

95% Confidence interval: [.00061,.001075]

Confidence interval does not contain zero, so conclude that  $\beta \neq 0$ .

Alternatively:

t-ratio is 7.227937. Is this big?

Yes, the P-value is  $5.5 \times 10^{-10}$  which is much less than .05.

Hence, we conclude again that  $\beta \neq 0$ .

## Jargon

- “The coefficient on population density is significantly different from zero.”
- “Population density has statistically significant explanatory power for deforestation.”
- “The hypothesis that  $\beta = 0$  can be rejected at the 5% significance level.”

## Testing on $R^2$ : The F-statistic

- Test whether  $R^2=0$  (i.e. whether X has any explanatory power)
- Note: In simple regression testing  $R^2=0$  and  $\beta=0$  are the same, but in multiple regression they will be different.
- F-statistic is a test statistic analogous to t-statistic (e.g. small values of it indicate  $R^2=0$ ).

$$F = \frac{(N-2)R^2}{(1-R^2)}$$

## Testing on $R^2$ : The F-statistic (cont.)

- For test with 5% level of significance:
- If P-value is  $> .05$  conclude  $R^2=0$ .
- If P-value is  $\leq .05$  conclude  $R^2 \neq 0$ .
- Excel calls the P-value for this test “Significance F”

## Example: The Regression of Deforestation on Population Density (cont.)

- P-value = Significance F =  $5.5 \times 10^{-10}$ .
- Since P-value  $< .05$  conclude  $R^2 \neq 0$ .
- Population density does have explanatory power for Y.

## Chapter 6: Multiple Regression

- Multiple regression same as simple regression except many explanatory variables:  $X_1, X_2, \dots, X_k$
- Intuition and derivation of multiple and simple regression very similar.
- We will emphasise only the few differences between simple and multiple regression.

## Example: Explaining House Prices

Data on N=546 houses sold in Windsor, Canada

Dependent variable:

Y = sales price of house

Explanatory variables:

$X_1$  = lot size of property (in square feet)

$X_2$  = number of bedrooms

$X_3$  = number of bathrooms

$X_4$  = number of storeys (excluding basement)

## OLS Estimation

- Multiple regression model:

$$Y_i = \alpha + \beta_1 X_{1i} + \dots + \beta_k X_{ki} + e_i$$

- OLS estimates:  $\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2, \dots, \hat{\beta}_k$

- Minimise Sum of Squared Residuals:

$$SSR = \sum (Y_i - \hat{\alpha} - \hat{\beta}_1 X_{1i} - \dots - \hat{\beta}_k X_{ki})^2$$

- Solution to minimisation problem: A mess
- Excel will calculate OLS estimates

## Statistical Aspects of Multiple Regression

- Largely the same as for multiple regression.
- Formulae of Chapter 5 have only minor modifications.
- $R^2$  still a measure of fit with same interpretation (although now it is no longer simply the correlation between Y and X squared).

## Statistical Aspects of Multiple Regression (cont.)

- Can test  $R^2=0$  in same manner as for simple regression.
- If you find  $R^2 \neq 0$  then you conclude that the explanatory variables *together* provide significant explanatory power (Note: this does not necessarily mean each individual explanatory variable is significant).
- Confidence intervals can be calculated for each *individual* coefficient as before.
- Can test  $\beta_j=0$  for each *individual* coefficient ( $j=1,2,\dots,k$ ) as before.

## Interpreting OLS Estimates in Multiple Regression Model

### Mathematical Intuition

- Total vs. partial derivative
- Simple regression:  $\frac{dY}{dX} = \beta$
- Multiple Regression:  $\frac{\partial Y}{\partial X_j} = \beta_j$

## Interpreting OLS Estimates in Multiple Regression Model (cont.)

### Verbal intuition

- $\beta_j$  is the marginal effect of  $X_j$  on  $Y$ , *ceteris paribus*
- $\beta_j$  is the effect of a small change in the  $j$ th explanatory variable on the dependent variable, *holding all the other explanatory variables constant*.

## Example: Explaining House Prices (cont.)

	Coeff.	St.Err	t-Stat	P-val.	Lower 95%	Upper 95%
Interc.	-4010	3603	-1.113	0.266	-11087	3068
Size	5.429	0.369	14.703	2.E-41	4.704	6.155
Bed.	2825	1215	2.325	0.020	438.3	5211
Bath.	17105	1734	9.862	3.E-21	13698	20512
Storeys	7635	1008	7.574	1.E-13	5655	9615

- $R^2=.54$  and the P-value for testing  $R^2=0$  (which is labelled "Significance F" by Excel) is 1.18E-88.
- Fitted regression line:

$$\hat{Y} = -4010 + 5.429X_1 + 2825X_2 + 17105X_3 + 7635X_4$$

## Example: Explaining House Prices (cont.)

Since  $\hat{\beta}_1 = 5.43$ :

- An extra square foot of lot size will tend to add \$5.43 onto the price of a house, *ceteris paribus*.
- For houses with the same number of bedrooms, bathrooms and storeys, an extra square foot of lots size will tend to add \$5.43 onto the price of a house.
- If we compare houses with the same number of bedrooms, bathrooms and storeys, those with larger lots tend to be worth more. In particular, an extra square foot in lot size is associated with an increased price of \$5.43.

## Example: Explaining House Prices (cont.)

Since  $\hat{\beta}_2 = 2,824.61$ :

- Adding one bedroom to your house will tend to increase its value by \$2,824.61, *ceteris paribus*.
- If we consider houses with comparable lot sizes and numbers of bathrooms and storeys, then those with an extra bedroom tend to be worth \$2,824.61 more.

## Pitfalls of Using Simple Regression in a Multiple Regression Context

- In multiple regression above, coefficient on number of bedrooms was 2,824.61.
- A simple regression of  $Y =$  house price on  $X =$  number of bedrooms yields a coefficient estimate of 13,269.98.
- Why are these two coefficients on the same explanatory variable so different? i.e. 13,269.98 >>> 2,824.61.

Answer 1: They just come from two different regressions which control for different explanatory variables (different *ceteris paribus* conditions).

## Pitfalls of Using Simple Regression in a Multiple Regression Context (cont.)

### Answer 2:

- Imagine a friend asked: “I have 2 bedrooms and I am thinking of building a third, how much will it raise the price of my house?”
- Simple regression: “Houses with 3 bedrooms tend to cost \$13,269.98 more than houses with 2 bedrooms”
- Does this mean adding a 3<sup>rd</sup> bedroom will tend to raise price of house by \$13,269.98? Not necessarily, other factors influence house prices.
- Houses with three bedrooms also tend to be desirable in other ways (e.g. bigger, with larger lots, more bathrooms, more storeys, etc.). Call these “good houses”.
- Simple regression notes “good houses” tend to be worth more than others.

## Pitfalls of Using Simple Regression in a Multiple Regression Context (cont.)

- Number of bedrooms is acting as a proxy for all these “good house” characteristics and hence its coefficient becomes very big (13,269.98) in simple regression.
- Multiple regression can estimate separate effects due to lot size, number of bedroom, bathrooms and storeys.
- Tell your friend: “Adding a third bedroom will tend to raise your house price by \$2,824.61”.
- Multiple regressions which contains all (or most) of house characteristics will tend to be more reliable than simple regression which only uses one characteristic.

## Pitfalls of Using Simple Regression in a Multiple Regression Context (cont.)

### Statistical evidence:

### Correlation matrix:

	Sale Price	Lot size	#bed	#bath	#storey
Sale price	1				
Lot size	0.5358	1			
#bed	0.3664	0.1519	1		
#bath	0.5167	0.1938	0.3738	1	
#storeys	0.4212	0.0837	0.4080	0.3241	1

- Positive correlations between explanatory variables indicate that houses with more bedrooms also tend to have larger lot size, more bathrooms and more storeys.

## Omitted Variable Bias

“Omitted variable bias” is a statistical term for these issues.

IF

1. We exclude explanatory variables that should be present in the regression,

AND

2. these omitted variables are correlated with the included explanatory variables,

THEN

3. the OLS estimates of the coefficients on the included explanatory variables will be biased.

## Omitted Variable Bias (cont.)

### Example:

- Simple regression used  $Y =$  house prices and  $X =$  number of bedrooms.
- Many important determinants of house prices omitted.
- Omitted variables were correlated with number of bedrooms. Hence, the OLS estimate from the simple regression  $\hat{\beta} = 13,269.98$  was biased.

## Practical Advice for Selecting Explanatory Variables

- Include (insofar as possible) all explanatory variables which you think might possibly explain your dependent variable. This will reduce the risk of omitted variable bias.
- However, including irrelevant explanatory variables reduces accuracy of estimation and increases confidence intervals. So do t-tests to decide whether variables are significant. Run a new regression omitting the explanatory variables which are not significant.

## Multicollinearity

- **Intuition:** If explanatory variables are highly correlated with one another then regression model has trouble telling which individual variable is explaining Y.
- **Symptom:** Individual coefficients may look insignificant, but regression as a whole may look significant (e.g.  $R^2$  big, F-stat big).
- Looking at a correlation matrix for explanatory variables can often be helpful in revealing extent and source of multicollinearity problem.

## Multicollinearity (cont.)

### Example:

Y = exchange rate

Explanatory variable(s) = interest rate

$X_1$  = bank prime rate  
 $X_2$  = Treasury bill rate

Using both  $X_1$  and  $X_2$  will probably cause multicollinearity problem

Solution: Include either  $X_1$  or  $X_2$  but not both.

In some cases this “solution” will be unsatisfactory if it causes you to drop out explanatory variables which economic theory says should be there.

## Example: Multicollinearity Illustrated using Artificial Data

True Model:

$$Y = .5X_1 + 2X_2 + e$$

Correlation between  $X_1$  and  $X_2 = .98$

	Coeff.	St. Error	t-Stat	P-val.	Lower 95%	Upper 95%
Inter.	.1662	.1025	1.579	.1211	-.0456	.3780
X1	2.084	.9529	2.187	.0338	.1667	4.001
X2	.1478	.9658	.1530	.8790	-1.795	2.091

- $R^2 = .76$
- P-value for testing  $R^2 = 0$  is  $1.87E-15$ .
- We want coefficient estimates of roughly .5 and 2 – this is not what we are getting. Plus  $X_2$  is not significant.

## Example: Multicollinearity Illustrated using Artificial Data (cont.)

Drop  $X_2$  and re-run the regression:

	Coeff.	St. Error	t-Stat	P-val.	Lower 95%	Upper 95%
Inter.	.1667	.1041	1.601	.1160	-.0427	.3761
X1	2.227	.1788	12.454	1.E-16	1.867	2.586

- $R^2 = .76$
- P-value for testing  $R^2 = 0$  is  $1.2E-16$ .
- Coefficient on X1 is significant, but is nowhere near .5!

## Chapter 7: Regression with Dummy Variables

- Dummy variable is either 0 or 1.
- Use to turn qualitative (Yes/No) data into 1/0.

### Example: Explaining House Prices

Dependent variable: Price of house

Explanatory variables:

- $D_1 = 1$  if the house has a driveway (=0 if it does not).
- $D_2 = 1$  if the house has a recreation room (=0 if not).
- $D_3 = 1$  if the house has a basement (=0 if not).
- $D_4 = 1$  if the house has gas central heating (=0 if not).
- $D_5 = 1$  if the house has air conditioning (=0 if not).

## Analysis of Variance

- Analysis of Variance = ANOVA
- Statistical technique commonly used in many fields (but not much used in economics).
- ANOVA is just a special case of regression with dummy variables.
- Regression with dummy variables is much more powerful and general tool.

### Simple Regression with a Dummy Variable

$$Y = \alpha + \beta D + e$$

- OLS estimation, confidence intervals, testing, etc. carried out in standard way.
- Interpretation a little different.

### Simple Regression with a Dummy Variable

- Fitted value for  $i^{\text{th}}$  observation (point on regression line):

$$\hat{Y}_i = \hat{\alpha} + \hat{\beta} D_i$$

- Since  $D_i = 0$  or 1 either

$$\hat{Y}_i = \hat{\alpha}$$

or

$$\hat{Y}_i = \hat{\alpha} + \hat{\beta}$$

### Example: Explaining house prices (continued)

- Regress  $Y$  = house price on  $D$  = dummy for air conditioning (=1 if house has air conditioning, = 0 otherwise).

- Result:

$$\hat{\alpha} = 59,885$$

$$\hat{\beta} = 25,996$$

$$\hat{\alpha} + \hat{\beta} = 85,881$$

- Average price of house with air conditioning is \$85,881
- Average price of house without air conditioning is \$59,885

### Multiple Regression with Dummy Variables

$$Y = \alpha + \beta_1 D_1 + \dots + \beta_k D_k + e$$

#### Example: Explaining house prices (continued)

Regress  $Y$  = house price on  $D_1$  = driveway dummy and  $D_2$  = rec room dummy

Four types of houses:

1. Houses with a driveway and a rec room ( $D_1=1, D_2=1$ )
2. Houses with a driveway but no rec room ( $D_1=1, D_2=0$ )
3. Houses with a rec room but no driveway ( $D_1=0, D_2=1$ )
4. Houses with no driveway and no rec room ( $D_1=0, D_2=0$ )

### Example: Explaining house prices (continued)

	Coeff.	St. Error	t Stat	P-value	Lower 95%	Upper 95%
Inter.	47099.1	2837.6	16.60	2.E-50	41525	52673
D1	21159.9	3062.4	6.91	1.E-11	15144	27176
D2	16023.7	2788.6	5.75	1.E-08	10546	21502

1. If  $D_1=1$  and  $D_2=1$ , then

$$\hat{Y} = \hat{\alpha} + \hat{\beta}_1 + \hat{\beta}_2 = 47,099 + 21,160 + 16,024 = 84,283$$

“The average price of houses with a driveway and rec room is \$84,283”.

### Example: Explaining house prices (continued)

2. If  $D_1 = 1$  and  $D_2=0$ , then

$$\hat{Y} = \hat{\alpha} + \hat{\beta}_1 = 47,099 + 21,160 = 68,259$$

“The average price of houses with a driveway but no rec room is \$68,259”.

3. If  $D_1=0$  and  $D_2=1$ , then

$$\hat{Y} = \hat{\alpha} + \hat{\beta}_2 = 47,099 + 16,024 = 63,123$$

“The average price of houses with a rec room but no driveway is \$63,123”.

4. If  $D_1=0$  and  $D_2=0$ , then

$$\hat{Y} = \hat{\alpha} = 47,099$$

“The average price of houses with no driveway and no rec room is \$47,099”.

## Multiple Regression with Dummy and non-Dummy Explanatory Variables

$$Y = \alpha + \beta_1 D + \beta_2 X + e$$

### Example: Explaining house prices (continued)

Regress Y = house price on D = air conditioning dummy and X = lot size.

OLS estimates:

$$\begin{aligned}\hat{\alpha} &= 32,693 \\ \hat{\beta}_1 &= 20,175 \\ \hat{\beta}_2 &= 5.64\end{aligned}$$

## Example: Explaining house prices (continued)

- For houses with an air conditioner D = 1 and

$$\hat{Y}_i = 52,868 + 5.64 X_i$$

- For houses without an air conditioner D=0 and

$$\hat{Y}_i = 32,693 + 5.64 X_i$$

- Two different regression lines depending on whether the house has an air conditioner or not.
- Two lines have different intercepts but same slope (i.e. same marginal effect)

## Example: Explaining house prices (continued)

Verbal ways of expressing OLS results:

- “An extra square foot of lot size will tend to add \$5.64 onto the price of a house” (Note: no *ceteris paribus* qualifications to statement since marginal effect is same for houses with and without air conditioners)
- “Houses with air conditioners tend to be worth \$20,175 more than houses with no air conditioners, *ceteris paribus*” (Note: Here we do have *ceteris paribus* qualification)
- “If we consider houses with similar lot sizes, those with air conditioners tend to be worth an extra \$20,175”

## Another House Price Regression

$$Y = \alpha + \beta_1 D_1 + \beta_2 D_2 + \beta_3 X_1 + \beta_4 X_2 + e.$$

- Regress Y = house price on D<sub>1</sub> = dummy variable for driveway, D<sub>2</sub> = dummy variable for rec room, X<sub>1</sub> = lot size and X<sub>2</sub> = number of bedrooms
- OLS estimates:

$$\begin{aligned}\hat{\alpha} &= -2736 \\ \hat{\beta}_1 &= 12,598 \\ \hat{\beta}_2 &= 10,969 \\ \hat{\beta}_3 &= 5.197 \\ \hat{\beta}_4 &= 10,562\end{aligned}$$

## Another House Price Regression (cont.)

1. If  $D_1=1$  and  $D_2=1$ , then

$$\hat{Y} = 20,831 + 5.197X_1 + 10,562X_2.$$

This is the regression line for houses with a driveway and rec room.

2. If  $D_1=1$  and  $D_2=0$ , then

$$\hat{Y} = 9,862 + 5.197 \times X_1 + 10,562 \times X_2.$$

This is the regression line for houses with a driveway but no rec room.

## Another House Price Regression (cont.)

3. If  $D_1=0$  and  $D_2=1$ , then

$$\hat{Y} = 8,233 + 5.197X_1 + 10,562X_2.$$

This is the regression line for houses with a rec room but no driveway.

4. If  $D_1=0$  and  $D_2=0$ , then

$$\hat{Y} = -2,736 + 5.197X_1 + 10,562X_2.$$

This is the regression line for houses with no driveway and no rec room.

## Another House Price Regression (cont.)

Examples of verbal statements:

- “Houses with driveways tend to be worth \$12,598 more than similar houses with no driveway.”
- “If we consider houses with the same number of bedrooms, then adding an extra square foot of lot size will tend to increase the price of a house by \$5.197.”
- “An extra bedroom will tend to add \$10,562 to the value of a house, *ceteris paribus*”

## Interacting Dummy and Non-Dummy Variables

$$Y = \alpha + \beta_1 D + \beta_2 X + \beta_3 Z + e.$$

where

$$Z = D \times X.$$

- $Z$  is either 0 (for observations with  $D=0$ ) or  $X$  (for observations with  $D=1$ )
- If  $D=1$  then  $\hat{Y} = (\hat{\alpha} + \hat{\beta}_1) + (\hat{\beta}_2 + \hat{\beta}_3)X$ .
- If  $D=0$ , then  $\hat{Y} = \hat{\alpha} + \hat{\beta}_2 X$ .
- Two different regression lines corresponding to  $D=0$  and  $D=1$  exist and have different intercepts and slopes.
- The marginal effect of  $X$  on  $Y$  is different for  $D=0$  and  $D=1$ .

## Yet Another House Price Example

- Regress  $Y$  = house price on  $D$  = air conditioner dummy,  $X$  = lot size and  $Z = D \times X$

- OLS estimates:

$$\hat{\alpha} = 35,684$$

$$\hat{\beta}_1 = 7,613$$

$$\hat{\beta}_2 = 5.02$$

$$\hat{\beta}_3 = 2.25.$$

- The marginal effect of lot size on housing is 7.27 for houses with air conditioners and only \$5.02 for houses without.
- Increasing lot size will tend to add more to the value of a house if it has an air conditioner than if it does not.

## Working with Dummy Dependent Variables

### Example:

Dependent variable is a transport choice.

1 = "Yes I take my car to work"

0 = "No I do not take my car to work"

- We will not discuss this case.
- Note only the following points:
  1. There are some problems with OLS estimation. But OLS estimation might be adequate in many cases.
  2. Better estimation methods are "Logit" and "Probit". Excel cannot easily do these, so you will need a different software package.

## Chapter 9: Univariate Time Series Analysis

- In previous chapter we discussed distributed lag models. These can be misleading if:
  1. The dependent variable  $Y_t$  depends on *lags of the dependent variable* as well, possibly, as  $X_t, X_{t-1}, \dots, X_{t-q}$ .
  2. The variables are nonstationary.
- In this chapter and the next, we develop tools for dealing with both issues and define what we mean by “nonstationary”.
- To simplify the analysis, focus solely on one time series,  $Y$ . Hence, *univariate time series analysis*.
- It is important to understand the properties of each individual series before proceeding to regression modelling involving several series.

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## Example: log of US personal income from 1954Q1 through 1994Q4.

See Figure 9.1

### Aside on logs

- It is common to take the natural logarithm of time series which are growing over time (i.e. work with  $\ln(Y)$  instead of  $Y$ ). Why?
- A time series graph of  $\ln(Y)$  will often approximate a straight line.
- In regressions with logged variables coefficients can be interpreted as elasticities.
- $\ln(Y_t) - \ln(Y_{t-1})$  is (approximately) the percentage change in  $Y$  between period  $t-1$  and  $t$ .

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### Example: US personal income (cont.)

- Note trend behaviour of personal income series.
- Many macroeconomic time series exhibit such trends.

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### Differencing

$$\Delta Y_t = Y_t - Y_{t-1}$$

- $\Delta Y_t$  measures the change (or growth) in  $Y$  between periods  $t-1$  and  $t$ .
- If  $Y_t$  is the log of a variable, then  $\Delta Y_t$  is the percentage change.
- $\Delta Y_t$  is the difference of  $Y$  (or first difference).
- $\Delta Y_t$  is often called “delta  $Y$ ”.

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## Example: US personal income (cont.)

See Figure 9.2.

- $\Delta Y$  = % change in personal income
- Not trending, very erratic.
- The differences of many macroeconomic time series have such properties.

## The Autocorrelation Function

- Correlation between  $Y$  and lags of itself shed important light of the properties of  $Y$ .
- Relates to the idea of a trend (discussed above) and nonstationarity (not discussed yet).

Example:  $Y$  = US personal income

- Correlation between  $Y_t$  and  $Y_{t-1}$  is .999716!
- Correlation between  $\Delta Y_t$  and  $\Delta Y_{t-1}$  is -.00235.
- These are *autocorrelations* (i.e. correlations between a variable and lags of itself).

## The Autocorrelation Function: Notation

- $r_1$  = correlation between  $Y$  and  $Y$  lagged one period.
- $r_p$  = correlation between  $Y$  and  $Y$  lagged  $p$  periods.
- *Autocorrelation function* treats  $r_p$  as a function of  $p$ .

## Example: US personal income (cont.)

### Autocorrelation functions of $Y$ and $\Delta Y$

Lag length (p)	Personal Income	Change in Pers. Income
1	.9997	-.0100
2	.9993	.0121
3	.9990	.1341
4	.9986	.0082
5	.9983	-.1562
6	.9980	.0611
7	.9978	-.0350
8	.9975	-.0655
9	.9974	.0745
10	.9972	.1488
11	.9969	.0330
12	.9966	.0363

- $Y$  is highly correlated with lags of itself, but the change in  $Y$  is not.
- Information could also be presented on bar charts. See Figures 9.3 and 9.4.

## Autocorrelation: Intuition

- **Y is highly correlated over time.  $\Delta Y$  does not exhibit this property.**
- **If you knew past values of personal income, you could make a very good estimate of what personal income was this quarter. However, knowing past values of the change in personal income will not help you predict the change in personal income this quarter.**
- **Y “remembers the past”.  $\Delta Y$  does not.**
- **Y is a nonstationary series while  $\Delta Y$  is stationary. (Note: These words not formally defined yet.)**

## The Autoregressive Model

- **Previous discussion has focussed on graphs and correlations, now we go on to regression.**

- **Autoregressive model of order 1 is written as AR(1) and given by:**

$$Y_t = \alpha + \phi Y_{t-1} + e_t$$

- **Figures 9.5, 9.6 and 9.7 indicate the types of behaviour that this model can generate.**
- **$\phi = 1$  generates trending behaviour typical of macro time series.**
- **$\phi = 0$  looks more like change in macro time series.**

## Nonstationary vs. Stationary Time Series

- **Formal definitions require difficult statistical theory. Some intuition will have to suffice.**
- **“Nonstationary” means “anything which is not stationary”.**
- **Focus on a case of great empirical relevance: unit root nonstationarity.**

## Ways of Thinking about Whether Y is Stationary or has a Unit Root

1. **If  $\phi = 1$ , then Y has a unit root. If  $|\phi| < 1$  then Y is stationary.**
2. **If Y has a unit root then its autocorrelations will be near one and will not drop much as lag length increases.**
3. **If Y has a unit root, then it will have a long memory. Stationary time series do not have long memory.**
4. **If Y has a unit root then the series will exhibit trend behaviour.**
5. **If Y has a unit root, then  $\Delta Y$  will be stationary. Hence, series with unit roots are often referred to as *difference stationary*.**

## More on the AR(1) Model

$$Y_t = \alpha + \phi Y_{t-1} + e_t$$

- Can rewrite as:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + e_t$$

$$\text{where } \rho = \phi - 1$$

- If  $\phi = 1$  (unit root) then  $\rho = 0$  and:

$$\Delta Y_t = \alpha + e_t$$

- Intuition: if Y has a unit root, can work with differenced data --- differences are stationary.

## More on the AR(1) Model

- Test if  $\rho = 0$  to see if a unit root is present.
- $-1 < \phi < 1$  is equivalent to  $-2 < \rho < 0$ . This is called the *stationarity condition*.

### Aside: The Random Walk Model:

$$Y_t = Y_{t-1} + e_t$$

- This is thought to hold for many financial variables such as stock prices, exchange rates.
- Intuition: Changes in Y are unpredictable, so no arbitrage opportunities for investors.

## Extensions of the AR(1) Model

- AR(p) model:

$$Y_t = \alpha + \phi_1 Y_{t-1} + \dots + \phi_p Y_{t-p} + e_t,$$

- Properties similar to the AR(1) model.

- Alternative way of writing AR(p) model:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \dots + \gamma_{p-1} \Delta Y_{t-p+1} + e_t.$$

- Coefficients in this alternative regression ( $\rho, \gamma_1, \dots, \gamma_{p-1}$ ) are simple functions of  $\phi_1, \dots, \phi_p$ .

## The AR(p) Model

- AR(p) is in the form of a regression model.
- $\rho=0$  implies that the time series Y contains a unit root (and  $-2 < \rho < 0$  indicates stationarity).
- If a time series contains a unit root then a regression model involving only  $\Delta Y$  is appropriate (i.e. if  $\rho = 0$  then the term  $Y_{t-1}$  will drop out of the equation).
- “If a unit root is present, then you can difference the data to induce stationarity.”

## More Extensions: Adding a Deterministic Trend

- Consider the following model:

$$Y_t = \alpha + \delta t + e_t.$$

- The term  $\delta t$  is a *deterministic trend* since it is an exact (i.e. deterministic) function of time.
- Unit root series contain a so-called *stochastic trend*.
- Combine with the AR(1) model to obtain:

$$Y_t = \alpha + \phi Y_{t-1} + \delta t + e_t.$$

- Can generate behaviour that looks similar to unit root behaviour even if  $|\phi| < 1$ . (i.e. even if they are stationary).
- See Figure 9.8.

## Summary

- The nonstationary time series variables on which we focus are those containing a unit root. These series contain a stochastic trend. If we difference these time series, the resulting time series will be stationary. For this reason, they are also called difference stationary.
- The stationary time series on which we focus have  $-2 < \rho < 0$ . But these series may exhibit trend behaviour through the incorporation of a deterministic trend. If this occurs, they are also called trend stationary.

## AR(p) with Deterministic Trend Model

- Most general model we use:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \dots + \gamma_p \Delta Y_{t-p} + \delta t + e_t.$$

- Why work with this form of the model?

- A unit root is present if  $\rho = 0$ . Easy to test.
- The specification is less likely to run into multicollinearity problems. Remember: in macroeconomics we often find  $Y$  is highly correlated with lags of itself but  $\Delta Y$  is not.

## Estimation of the AR(p) with Deterministic Trend Model

- OLS can be done in usual way.

Example:  $Y$  = US personal income

- $\Delta Y$  is the dependent variable in the regression below.

### AR(4) with Deterministic Trend Model

	Coeff.	St. Error	t-Stat	P-val	Lower 95%	Upper 95%
Inter.	.138	.108	1.279	.203	-.075	.351
$Y_{t-1}$	-.018	.015	-1.190	.236	-.049	.012
$\Delta Y_{t-1}$	-.017	.081	-.217	.829	-.177	.142
$\Delta Y_{t-2}$	.014	.081	.172	.863	-.145	.173
$\Delta Y_{t-3}$	.130	.080	1.627	.106	-.028	.288
time	.00012	.00012	.955	.341	-.00013	.00037

## Testing in AR(p) with Deterministic Trend Model

- For everything except  $\rho$ , testing can be done in usual way using t-statistics and P-values.
- Hence, can use standard tests to decide whether to include deterministic trend.

### Lag length selection

- A common practice: begin with an AR(p) model and look to see if the last coefficient,  $\gamma_p$  is significant. If not, estimate an AR(p-1) model and see if  $\gamma_{p-1}$  is significant. If not, estimate an AR(p-2), etc.

## Example: Y = US personal income

- Sequential testing strategy leads us to drop the deterministic trend and go all the way back to a model with one lag, an AR(1).
- $\Delta Y$  is the dependent variable in the regression.

	Coeff.	St. Error	t-Stat	P-val	Lower 95%	Upper 95%
Inter.	.039	.014	2.682	.008	.010	.067
$Y_{t-1}$	-.004	.002	-2.130	.035	-.0077	-.0003

## Testing for a Unit Root

- You might think you can test  $\rho = 0$  in the same way (i.e. look at P-value and, if it is less than .05, reject the unit root hypothesis, if not accept the unit root).
- **THIS IS INCORRECT!**
- Justification: Difficult statistics.
- Essentially: Excel gets the t-statistic correct, but the P-value is wrong.
- A correct test is the Dickey-Fuller Test, which uses the t-statistic and compares it to a critical value.

## Practical Advice on Unit Root Testing

- Learn more econometrics using another textbook.
- Use a computer software package which is suitable for time series.
- Use the following rough rule of thumb which should be okay if sample size is moderately large (e.g.  $T > 50$ ).

## Testing for a Unit Root: An Approximate Strategy

1. Use the sequential testing strategy outlined above to estimate the AR(p) with deterministic trend model. Record the t-stat corresponding to  $\rho$  (i.e. the coefficient on  $Y_{t-1}$ ).
2. If the final version of your model includes a deterministic trend, the Dickey-Fuller critical value is approximately  $-3.45$ . If the t-stat on  $\rho$  is more negative than  $-3.45$ , reject the unit root hypothesis and conclude that the series is stationary. Otherwise, conclude that the series has a unit root.
3. If the final version of your model does not include a deterministic trend, the Dickey-Fuller critical value is approximately  $-2.89$ . If the t-stat on  $\rho$  is more negative than this, reject the unit root hypothesis and conclude that the series is stationary. Otherwise, conclude that the series has a unit root.

## Example: Y = US personal income

- Sequential testing strategy leads from:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \dots + \gamma_p \Delta Y_{t-p} + \delta + e_t,$$

to

$$\Delta Y_t = \alpha + \rho Y_{t-1} + e_t$$

	Coeff.	St. Error	t-Stat	P-val	Lower 95%	Upper 95%
Inter.	.039	.014	2.682	.008	.010	.067
$Y_{t-1}$	-.004	.002	-2.130	.035	-.0077	-.0003

- t-stat on  $\rho$  is  $-2.13$ , which is not more negative than  $-2.89$ . Hence, we can accept the hypothesis that personal income does contain a unit root.

**College Town**

FCI owns 10 apartment buildings in a college town, which it rents exclusively to students. Each apartment building contains 100 rental units, but the owner is having cash flow problems due to an average vacancy rate of nearly 50 percent. The apartments in each building have comparable floor plans, but some buildings are closer to campus than others. The owner of FCI has data from last year on the number of apartments rented, the rental price (in dollars), and the amount spent on advertising (in hundreds of dollars) at each of the 10 apartments. These data, along with the distance (in miles) from each apartment building to campus, are presented in rows 1 through 11 of Table 3–9. The owner regressed the quantity demanded of apartments on price, advertising, and distance. The results of the regression are reported in rows 16 through 22 of Table 3–9. What is the estimated demand function for FCI’s rental units? If FCI raised rents at one complex by \$100, what would you expect to happen to the number of units rented? If FCI raised rents at an average apartment building, what would happen to FCI’s total revenues? What inferences should be drawn from this analysis?

**Table 3–9** Input and Output from a Multiple Regression

	A	B	C	D	E	F	G
1	Observation	Quantity	Price	Advertising	Distance		
2	1	28	250	11	12		
3	2	69	400	24	6		
4	3	43	450	15	5		
5	4	32	550	31	7		
6	5	42	575	34	4		
7	6	72	375	22	2		
8	7	66	375	12	5		
9	8	49	450	24	7		
10	9	70	400	22	4		
11	10	60	375	10	5		
12	<b>Average</b>	53.10	420.00	20.50	5.70		
13							
14							
15							
16	<b>Regression Statistics</b>						
17							
18	Multiple R	0.89					
19	R-Square	0.79					
20	Adjusted R-Square	0.69					
21	Standard Error	9.18					
22	Observations	10.00					
23							

Quelle: Baye, Michael: Managerial Economics and Business Strategy, Mc Graw 2003, p 101

Analyse the data in Excel:

- **Describe the relation between the quantity and the independent variables in bivariate correlations and regressions. What do we learn?**
- Continue the multiple regression (already started above).
  - Which are the numerical results?
  - What do we learn? Interpret the results (for example answering the questions above); what is your recommendation for FCI?