Prof. Dr. Peter Schmidt Summer Term 2024


IGC
International Graduate Center
Hochschule Bremen
City University of Applied Sciences

## Economics \& Statistics

용: (0421) 5905-4691 Fax: (0421) 5905-4862
Peter.Schmidt@hs-bremen.de schmidt-bremen.de -> MBA QM

## Quantitative Methods

## Executive MBA

- Content
- References
- Material


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## The Business Research Process



Source: Cooper, D. and Schindler, P.S.: "Business Research Methods"

## Chapter 1

research in business


## Why Study Business Research?

Business research provides information to guide business decisions


## What's Changing in Business

 that Influences Research

## Learning Objectives

Understand . .

- What business research is and how it differs from business decision support systems and business intelligence systems.
- Trends affecting business research and the emerging hierarchy of business decision makers.
- The distinction between good business research
and research that falls short of professional quality.
- The nature of the research process.


## Business Research

- A process of determining, acquiring, analyzing, synthesizing, and disseminating relevant business data, information, and insights to decision makers in ways that mobilize the organization to take appropriate business actions that, in turn, maximize business performance


## Computing Power and Speed



## Pull Quote

"Forward-thinking executives recognize that analytics may be the only true source of sustainable advantage since it empowers employees at all levels of an organization with information to help them make smarter decisions."

Wayne Eckerson,
director of research, business applications and architecture group, TechTarget

## The Research Process

Stage 1: Clarifying the
Research question
Stage 2: Proposing Research
Stage 3: Designing the
Research
Stage 4: Data Collection \&
Preparation
Stage 5: Data Analysis \&
Interpretation
Stage 6: Reporting the
Results

## Information Sources

## Decision Support Systems

- Numerous elements of data organized for retrieval and use in business decision making
- Stored and retrieved via

Intranets
Extranets

Business
Intelligence Systems

- Ongoing information collection
- Focused on events, trends in micro and macro-environments

Sources of Business Intelligence


Information Value Chain


Hierarchy of Business Decision Makers


## Chapter 2

ethics in business research


Types of Ethical Violations


## Research May Not Be Necessary

Can It Pass These Tests?

- Can information be applied to a critical decision?
- Will the information improve
managerial decision making?
- Are sufficient resources available?


## Ethical Treatment of Participants



## Unethical Behavior to Avoid




## Language of Research



## A Variable: Property Being Studied



## Learning Objectives

## Understand . . .

- The terminology used by professional researchers employing scientific thinking.
- What you need to formulate a solid research hypothesis.
- The need for sound reasoning to enhance research results.



## Types of Variables



## Language of Research



## Operational Definitions

How can we define the variable "class level of students"?

- Freshman
- <30 credit hours
- Sophomore
- 30-50 credit hours
- J unior
- 60-89 credit hours
- Senior
- > 90 credit hours


## Independent and Dependent Variable Synonyms

## Independent <br> Variable (IV)

- Predictor
- Presumed cause
- Stimulus
- Predicted from...
- Antecedent
- Manipulated


## Dependent

 Variable (DV)- Criterion
- Presumed effect
- Response
- Predicted to...
- Consequence
- Measured outcome



## The Role of Hypotheses



Relationships Among Variable Types


## Descriptive Hypothesis Formats

## Descriptive <br> Hypothesis

- In Detroit, our potato chip market share stands at 13.7\%.
- American cities are experiencing budget difficulties.


## Characteristics of Strong Hypotheses



Relationships Among Variable Types


## Relational Hypotheses Formats

## - Correlational

- Young women (under

35) purchase fewer units of our product than women who are older than 35.

The number of suits sold varies directly with the level of the business cycle.

## - Causal

- An increase in family income leads to an increase in the percentage of income saved.
- Loyalty to a grocery store increases the probability of purchasing that store's private brand products.


## The Role of Reasoning



## The Scientific Method



## Sound Reasoning



## Why Didn't Sales Increase?




## Deductive Reasoning



Chapter 4
THE RESEARCH PROCESS: AN OVERVIEW


Why is curiosity important?


## Inductive Reasoning

## - Why didn't sales increase during our

 promotional event?- Regional retailers did not have sufficient stock to fill customer requests during the promotional period
- A strike by employees prevented stock from arriving in time for promotion to be effective
- A hurricane closed retail outlets in the region for 10 days during the promotion


## Learning Objectives

## Understand ...

- That research is decision- and dilemma-centered.
- That the clarified research question is the result of careful exploration and analysis and sets the direction for the research project.


## Learning Objectives

## Understand . . .

- How value assessments and budgeting influence the process for proposing research, and ultimately, research design.
- What is included in research design, data collection, and data analysis.
- Research process problems to avoid.


Stage 4: Data Collection

Data Collection \& Preparation

## The Research Process

Stage 1: Clarifying the Research question
Stage 2: Proposing Research
Stage 3: Designing the Research
Stage 4: Data Collection \& Preparation
Stage 5: Data Analysis \& Interpretation
Stage 6: Reporting the Results


The Research Proposal


Stage 1: Clarifying the Research Question


Management-research question hierarchy process begins by identifying the management dilemma

Stage 3: Designing the Research


```
Data Analysis & Interpretation
```


## Parts of the Research Report



## Learning Objectives

## Understand . . .

- The basic stages of research design.
- The major descriptors of research design.
- The major types of research designs.
- The relationships that exist between variables in research design and the steps for evaluating those relationships.



## The Research Report Overview



## What Tools Are Used in Designing Research?



| viestones | Criteal Pathe |
| :---: | :---: |
| Proposal ipproval | Time to Completion |
| Rav repar compatea | 40 mymog tam |

Stage 6: Reporting the Results


Chapter 6
RESEARCH DESIGN: AN OVERVIEW


## Research Design Descriptors



The Time Dimension


## Causal Studies



## Degree of

Question Crystallization

- Exploratory Study
- Loose structure
- Expand
understanding
- Provide insight
- Develop hypotheses
- Formal Study
- Precise procedures
- Begins with hypotheses
- Answers research questions

Commonly Used Exploratory Techniques


## Descriptive Studies



Chapter 10
SURVEYS

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## Learning Objectives

Understand...

- The process for selecting the appropriate and optimal communication approach.
- Factors affect participation in communication studies.
- Sources of error in communication studies and how to minimize them.
- Major advantages and disadvantages of the three communication approaches.
- Why an organization might outsourcea communication study.


## Sources of Error



## Communication Approaches




## Participant Motivation



PicProfile: Methodology Trends



## Response Terms



## PicProfile: Declining Phone Response



## Pull Quote

"You're trying too hard to find a correlation here. You don't know these people, you don't know what they intended. You try to compile statistics and correlate them to a result that amounts to nothing more than speculation."

Marc Racicot, former governor of Montana and chairman of the Republican Party

## Characteristics of Measurement



## Chapter 11

MEASUREMENT


Types of Scales


## Learning Objectives

## Understand . . .

- The distinction between measuring objects, properties, and indicants of properties.
- The similarities and differences between the four scale types used in measurement and when each is used.
- The four major sources of measurement error.
- The criteria for evaluating good measurement.


## Measurement



## Levels of Measurement



Nominal Scales


## Levels of Measurement



## Ratio Scales



## Levels of Measurement



Ordinal Scales


## Levels of Measurement


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## Sources of Error



## Increasing Content Validity



Summary of Validity Estimates


## Evaluating Measurement Tools



Increasing Construct Validity

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## Validity Determinants



## Judging Criterion Validity



## Reliability Estimates




## Pull Quote

"No man learns to know his inmost nature by introspection, for he rates himself sometimes too low, and often too high, by his own measurement. Man knows himself only by comparing himself with other men; it is life that touches his genuine worth."

Johann Wolfgang von Goethe German writer, artist, politician (1749-1832)

## Response Types




## Number of Dimensions



## Learning Objectives

## Understand...

- The nature of attitudes and their relationship to behavior.
- The critical decisions involved in selecting an appropriate measurement scale.
- The characteristics and use of rating, ranking, sorting, and other preference scales.


## Selecting a Measurement Scale



## Balanced or Unbalanced

## How good an actress is Jennifer Lawrence?

| Very bad |
| :--- |
| Bad |
| Neither good nor bad |
| Good |
| Very good |

Poor Fair
Good Very good
Excellent

## Forced or Unforced Choices

How good an actress is Jennifer Lawrence？

| Very bad |  |
| :--- | :--- |
| Bad |  |
| Neither good nor bad |  |
| Good |  |
| Very good | Very bad  <br>  Bad <br> Neither good nor bad <br> Good <br> Very good <br> No opinion <br> Don＇t know$⿳ 亠 口$ |

Rater Errors


## Number of Scale Points

How good an actress is Jennifer Lawrence？
Very bad
Bad
Neither good nor bad
Good
Very good

Rater Errors


## Simple Category Scale



Multiple－Choice， Multiple－Response Scale




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## Learning Objectives

## Understand...

- The link forged between the management dilemma and the communication instrument by the management-research question hierarchy.
- The influence of the communication method on instrument design.
- The three general classes of information and what each contributes to the instrument.


## Learning Objectives

Understand . . .

- The influence of question content, question wording, response strategy, and preliminary analysis planning on question construction.
- Each of the numerous question design issues influencing instrument quality, reliability, and validity.
- The sources for measurement questions
- The importance of pretesting questions and instruments.


## Strategic Concerns in Instrument Design



## Question Categories

 and Structure

## Overall Flowchart for Instrument Design



Factors Affecting Respondent Honesty


## Engagement $=$ Convenience

"Participants are becoming more and more aware of the value of their time. The key to maintaining a quality dialog with them is to make it really convenient for them to engage, whenever and wherever they want."

Tom Anderson managing partner Anderson Analytics

Flowchart for Instrument Design Phase 1


Flowchart for Instrument Design Phase 2


## Question Content



Question Wording


Dichotomous Response Strategy


## Rating Response Strategy

|  | Strongly <br> influential | Somewhat <br> influential | Not at all <br> influential |
| :---: | :---: | :---: | :---: |
| Good academic <br> reputation <br> Enjoyable campus life | $\square$ | $\square$ | $\square$ |
| Many friends | $\square$ | $\square$ | $\square$ |
| High quality faculty | $\square$ | $\square$ | $\square$ |
| Semester calendar | $\square$ | $\square$ | $\square$ |

## Response Strategy



Multiple Choice Response Strategy


## Ranking



Free-Response Strategy


## Checklist Response Strategy



## Summary of Scale Types



Summary of Scale Types

| Type | Restrictions | Scale Items | Data Type |
| :---: | :---: | :---: | :---: |
| Rating Scales |  |  |  |
| Numerical | - Needs concepts with standardized meanings; <br> -Needs number anchors of the scale or end-points <br> - Score is a measurement of graphical space | One or many | Ordinal or Interval |
| $\begin{aligned} & \text { Multiple } \\ & \text { Rating List } \\ & \text { Scale } \end{aligned}$ | - Needs words that are opposites to anchor the end-points on the verbal scale | Up to 10 | Ordinal |
| $\begin{aligned} & \text { Fixed Sum } \\ & \text { Scale } \end{aligned}$ | -Participant needs ability to calculate total to some fixed number, often 100. | Two or more | Interval or Ratio |

## Internet Survey Scale Options



## Sources of Questions

- Handbook of Marketing Scales
- The Gallup Poll

Cumulative Index

- Measures of

Personality and
Social-Psychological Attitudes

- Measures of Political Attitudes
- Index to International Public Opinion
Sourcebook of Harris National Surveys Marketing Scales Handbook
American Social Attitudes Data Sourcebook

Summary of Scale Types


## Internet Survey Scale Options



Summary of Scale Types

| Type | Restrictions | Scale <br> Items | Data Type |
| :---: | :---: | :---: | :---: |
| Ranking Scales |  |  |  |
| Paired Comparison <br> Scale | Number is controlled by participant's <br> stamina and interest. | Up to 10 | Ordinal |
| Forced Ranking <br> Scale | - Needs mutually exclusive choices. | Up to 10 | Ordinal or <br> Interval |
| Comparative Scale | -Can use verbal or graphical scale. | Up to 10 | Ordinal |

## Internet Survey Scale Options



Guidelines for Question Sequencing


PicProfile: Branching Question


## Chapter 14

sAMPLing


## The Nature of Sampling



Snapshot: Mobile Questionnaires


## Learning Objectives

## Understand...

- The two premises on which sampling theory is based.
- The accuracy and precision for measuring sample validity.
- The five questions that must be answered to develop a sampling plan.


## Why Sample?



## When Is a Census Appropriate?



## Research Thought Leader

"Research that asks consumers what they did and why is incredibly helpful. Research that asks consumers what they are going to do can often be taken with a grain of salt."

Al Ries
author, co-founder, and chairman Ries \& Ries.

## Learning Objectives

## Understand . . .

- The two categories of sampling techniques and the variety of sampling techniques within each category.
- The various sampling techniques and when each is used.


## What Is a Valid Sample?



## When to Use Larger Sample?



## Stratified

## Advantages

- Control of sample size in strata
- Increased statistical efficiency
- Provides data to represent and analyze subgroups - Enables use of different methods in strata


## Types of Sampling Designs

| Element <br> Selection | Probability | Nonprobability |
| :---: | :---: | :---: |
| -Unrestricted | • Simple random | • Convenience |
| -Restricted | • Complex random | • Purposive |
|  | •Systematic | • Judgment |
|  | •Cluster | •Quota |
|  | •Stratified | •Snowball |
|  | •Double |  |

## Simple Random



## Cluster



## Steps in Sampling Design



## Systematic

## Advantages

- Simple to design
- Easier than simple random
- Easy to determine sampling distribution of mean or proportion


## Disadvantages

- Periodicity within population may skew sample and results
- Trends in list may bias results
- Moderate cost


## Stratified and Cluster Sampling

| Stratified |
| :--- |
| - Population divided into |
| few subgroups |
| - Homogeneity within |
| subgroups |
| - Heterogeneity between |
| subgroups |
| - Choice of elements from |
| within each subgroup |

## Cluster

- Population divided into many subgroups
- Heterogeneity within subgroups
- Homogeneity between subgroups
- Random choice of subgroups


## Area Sampling



## Nonprobability Sampling Methods



## Distributions



## Double Sampling



Nonprobability Samples


## Frequencies



Measures of Central Tendency


## Measures of Variability



## Appendix 14a

Determining Sample Size


Confidence Levels \& the Normal Curve


Summarizing Distribution Shape


## Random Samples



## Standard Errors

| Standard <br> Error <br> (Z score) | \% of Area | Approximate <br> Degree of <br> Confidence |
| :---: | :---: | :---: |
| 1.00 | 68.27 | $68 \%$ |
| 1.65 | 90.10 | $90 \%$ |
| 1.96 | 95.00 | $95 \%$ |
| 3.00 | 99.73 | $99 \%$ |

Symbols

| Variable | Population | Sample |
| :--- | :---: | :---: |
| Mean | $\mu$ | $\mathbf{x}$ |
| Proportion | $\Pi$ | p |
| Variance | $\sigma^{2}$ | $\mathbf{s}^{2}$ |
| Standard deviation | $\sigma$ | $-\mathbf{s}$ |
| Size | N | n |
| Standard error of the mean | $\sigma_{\mathrm{x}}$ | $\mathbf{S}_{\mathrm{x}}$ |
| Standard error of the proportion | $\sigma_{\mathrm{p}}$ | $\mathbf{s}_{\mathrm{p}}$ |

## Increasing Precision



## Central Limit Theorem



## Estimates of Dining Visits

| Confidence | $Z$ <br> score | \% of <br> Area | Interval Range <br> (visits per month) |
| :---: | :---: | :---: | :---: |
| $68 \%$ | 1.00 | 68.27 | $9.48-10.52$ |
| $90 \%$ | 1.65 | 90.10 | $9.14-10.86$ |
| $95 \%$ | 1.96 | 95.00 | $8.98-11.02$ |
| $99 \%$ | 3.00 | 99.73 | $8.44-11.56$ |

## Proxies of the Population Dispersion



## Learning Objectives

## Understand . .

- The nature and logic of hypothesis testing.
- A statistically significant difference
- The six-step hypothesis testing procedure.


## Calculating Sample Size for Questions involving Means



Metro U Sample Size for Proportions

| Steps | Information |
| :---: | :---: |
| Desired confidence level | $95 \%(z=1.96)$ |
| Size of the interval estimate | $\pm .10(10 \%)$ |
| Expected range in population | 0 to $100 \%$ |
| Sample proportion with given <br> attribute | $30 \%$ |
| Sample dispersion | $P q=.30(1-.30)=.21$ |
| Finite population adjustment | No |
| Standard error of the <br> proportion | $.10 / 1.96=.051$ |
| Sample size | $.21 /(.051)^{2}=81$ |

## Learning Objectives

## Understand...

- The differences between parametric and nonparametric tests and when to use each.
- The factors that influence the selection of an appropriate test of statistical significance.
- How to interpret the various test statistics

Metro U Sample Size for Means

| Steps | Information |
| :---: | :---: |
| Desired confidence level | $95 \%(z=1.96)$ |
| Size of the interval estimate | $\pm .5$ meals per month |
| Expected range in <br> population | 0 to 30 meals |
| Sample mean | 10 |
| Standard deviation | 4.1 |
| Need for finite population <br> adjustment | No |
| Standard error of the mean | $.5 / 1.96=.255$ |
| Sample size | $(4.1)^{2 /}(.255)^{2}=259$ |

## Chapter 17

hypothesis testing


## Hypothesis Testing



Hypothesis Testing Finds Truth
"One finds the truth by making a hypothesis and comparing the truth to the hypothesis."

David Douglass
physicist
University of Rochester


## Types of Hypotheses

## Null

- $\mathrm{H}_{0}: \mu=50 \mathrm{mpg}$
- $\mathrm{H}_{0}: \mu \leq 50 \mathrm{mpg}$
- $\mathrm{H}_{0}: \mu \geq 50 \mathrm{mpg}$


## Alternate

- $\mathrm{H}_{\mathrm{A}}: \mu \neq 50 \mathrm{mpg}$
- $\mathrm{H}_{\mathrm{A}}: \mu>50 \mathrm{mpg}$
- $\mathrm{H}_{\mathrm{A}}: \mu<50 \mathrm{mpg}$


## Statistical Procedures



## Approaches to Hypothesis Testing

## Classical statistics

- Objective view of probability
- Established
hypothesis is rejected or fails to be rejected
- Analysis based on sample data


## Bayesian statistics

Extension of classical approach

- Analysis based on sample data
Also considers established subjective probability estimates


## Two-Tailed Test of Significance




## Statistical Significance



## One-Tailed Test of Significance



## Decision Rule

Take no corrective action if the analysis shows that one cannot reject the null hypothesis.

## Critical Values

$$
\begin{aligned}
& Z=1.96 \text { (significance level }=.05 \text { ) } \\
& \bar{X}_{c}=\text { the critical value of the sample mean } \\
& \mu=\text { the population value stated in } H_{0}=50 \\
& \sigma_{X}=\text { the standard error of a distribution of means of samples of } 25
\end{aligned} \qquad \begin{aligned}
& Z=\frac{\bar{X}-\mu}{\sigma_{\bar{X}}} \\
&-1.96=\frac{\bar{X}_{c}-50}{2} 1.96=\frac{\bar{X}_{c}-50}{2} \\
& \bar{X}_{c}=46.08 \bar{X}_{c}=53.92
\end{aligned}
$$

## Probability of Making A Type II Error



Statistical Decisions


Probability of Making A Type I Error


Statistical Testing Procedures


Probability of Making a Type I Error


Factors Affecting Probability of Committing a $\beta$ Error


## Tests of Significance




## Recommended Statistical Techniques



## One-Sample t-Test Example

| Null | Ho: $=50 \mathrm{mpg}$ |
| :--- | :---: |
| Statistical test | t -test | \left\lvert\, | $.05, n=100$ |
| :--- |
| Significance level |
| Calculated value |
| Critical test value | | 1.786 |
| :---: |
| (from Appendix C, |
| Exhibit C-2) |\right.

Advantages of Nonparametric Tests


## Questions Answered by One-Sample Tests

Is there a difference between observed frequencies and the frequencies we would expect?

Is there a difference between observed and expected proportions?

Is there a significant difference between some measures of central tendency and the population parameter?

## One Sample Chi-Square Test Example

| Living Arrangement | Intend <br> to Join | Number <br> Interviewed | Percent <br> (no. interviewed/200) | Expected <br> Frequencies <br> (percentx <br> 60) |
| :--- | :---: | :---: | :---: | :---: |
| Dormfraternity | 16 | 90 | 45 | 27 |
| Apartmentrooming <br> house, nearby | 13 | 40 | 20 | 12 |
| Apartmentrooming <br> house, listant | 16 | 40 | 20 | 12 |
| Live at home <br> Total | 15 | 30 | 15 | 9 |

## How to Select a Test

## How many samples are involved?

If two or more samples: are the individual cases independent or related?

Is the measurement scale nominal, ordinal, interval, or ratio?

## Parametric Tests



## Two-Sample Parametric Tests

$$
\begin{array}{r}
Z=\frac{\left(\bar{X}_{1}-\bar{X}_{2}\right)-\left(\mu_{1}-\mu_{2}\right) 0}{\sqrt{\frac{S_{1}^{2}}{n_{1}}+\frac{S_{2}^{2}}{n_{2}}}} \\
t=\frac{\left(\bar{X}_{1}-\bar{X}_{2}\right)-\left(\mu_{1}-\mu_{2}\right) 0}{\sqrt{S_{p}^{2}\left(\frac{1}{n_{1}}+\frac{1}{n_{2}}\right)}}
\end{array}
$$

Two-Sample t-Test Example

|  | A Group | B Group |
| :---: | :---: | :---: |
| Average <br> hourly sales | $\mathrm{X}_{1}=$ | $\mathrm{X} 2=$ |
| Standard <br> deviation | $\mathrm{s}_{1}=225$ | $\mathrm{~s} 2=251,300$ |

## Two-Sample Chi-Square Example

| Null | There is no difference in <br> distribution channel for age <br> categories. |
| :--- | :---: |
| Statistical test | Chi-square |
| Significance level | .05 |
| Calculated value | 6.86, d.f. $=2$ |
| Critical test value | 5.99 <br> (from Appendix c, Exhibit c-3) |

## ANOVA Example

| Model Summary |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Source | d.f. | Sum of Squares | Mean Square | F Value | p Value |
| Model (airine) | 2 | 11644.033 | 5822.017 | 28.304 | 0.0001 |
| Residual (error) | 57 | 11724.550 | 205.694 |  |  |
| Total | 59 | 23368.583 |  |  |  |


|  | Means Table |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Count | Mean | Std. Dev. | Std. Error |
| Luthansa | 20 | 38.950 | 14.006 | 3.132 |
| Malaysia Airines | 20 | 58.900 | 15.089 | 3.374 |
| Cathay Pacific | 20 | 72.900 | 13.902 | 3.108 |
| All data are hypothe |  |  |  |  |

Two-Sample t-Test Example

| Null | Ho: A sales $=$ B sales |
| :--- | :---: |
| Statistical test | t-test |
| Significance level | .05 (one-tailed) |
| Calculated value | 1.97, d.f. $=20$ |
| Critical test value | 1.725 <br> (from Appendix C, Exhibit C-2) |

## SPSS Cross-Tabulation Procedure



Chi-Square





## ANOVA Example Continued

| Null | $\mu \mathrm{A1}=\mu \mathrm{A} 2=\mu \mathrm{A} 3$ |
| :---: | :---: |
| Statistical test | ANOVA and F ratio |
| Significance level | .05 |
| Calculated value | 28.304, d.f. $=2,57$ |
| Critical test value | 3.16 <br> (from Appendix C, Exhibit C-9) |

Two-Sample Nonparametric Tests: Chi-Square

|  | On-the-Job-Accident |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Yes | No | Row Total |
| smoker | Heavy Smoker |  | 1,2 <br> ¢ 7.7 <br> .8 | 16 |
|  | Moderate | $\begin{aligned} & \hline 2.1 \\ & 9 \\ & 7.73 \end{aligned}$ | 2,6 7.27 7.27 | 15 |
|  | oker | $\begin{gathered} 3,1 \\ 13 \\ 18.03 \end{gathered}$ | 3,2 .22 16.97 | 35 |
|  | Iumn Total | 34 | 32 | 66 |

## k-Independent-Samples Tests: ANOVA

Tests the null hypothesis that the means of three or more populations are equal.

One-way: Uses a single factor, fixed-effects model to compare the effects of a treatment or factor on a continuous dependent variable.

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## C H A P TER

## Introduction

There are several types of professional economists working in the world today. Academic economists in universities often derive and test theoretical models of various aspects of the economy. Economists in the civil service often study the merits and demerits of policies under consideration by government. Economists employed by a central bank often give advice on whether or not interest rates should be raised, while in the private sector, economists often predict future variables such as exchange rate movements and their effect on company exports.

For all of these economists, the ability to work with data is an important skill. To decide between competing theories, to predict the effect of policy changes, or to forecast what may happen in the future, it is necessary to appeal to facts. In economics, we are fortunate in having at our disposal an enormous amount of facts (in the form of "data") that we can analyze in various ways to shed light on many economic issues.
The purpose of this book is to present the basics of data analysis in a simple, nonmathematical way, emphasizing graphical and verbal intuition. It focusses on the tools that economists apply in practice (primarily regression) and develops computer skills that are necessary in virtually any career path that the economics student may choose to follow.
To explain further what this book does, it is perhaps useful to begin by discussing what it does not do. Econometrics is the name given to the study of quantitative tools for analyzing economic data. The field of econometrics is based on probability and statistical theory; it is a fairly mathematical field. This book does not attempt to teach probability and statistical theory. Neither does it contain much mathematical content. In both these respects, it represents a clear departure from traditional econometrics textbooks. Yet, it aims to teach most of the practical tools used by applied econometricians today.

Books that merely teach the student which buttons to press on a computer without providing an understanding of what the computer is doing, are commonly referred to as "cookbooks". The present book is not a cookbook. Some econometricians may interject at this point: "But how can a book teach the student to use the tools of econometrics, without teaching the basics of probability and statistics?" My answer is that much of what the econometrician does in practice can be understood intuitively, without resorting to probability and statistical theory. Indeed, it is a contention of this book that most of the tools econometricians use can be mastered simply through a thorough understanding of the concept of correlation, and its generalization, regression. If a student understands correlation and regression well, then he/she can understand most of what econometricians do. In the vast majority of cases, it can be argued that regression will reveal most of the information in a data set. Furthermore, correlation and regression are fairly simple concepts that can be understood through verbal intuition or graphical methods. They provide the basis of explanation for more difficult concepts, and can be used to analyze many types of economic data.
This book focusses on the analysis of economic data; it is not a book about collecting economic data. With some exceptions, it treats the data as given, and does not explain how the data is collected or constructed. For instance, it does not explain how national accounts are created or how labor surveys are designed. It simply teaches the reader to make sense out of the data that has been gathered.
Statistical theory usually proceeds from the formal definition of general concepts, followed by a discussion of how these concepts are relevant to particular examples. The present book attempts to do the opposite. That is, it attempts to motivate general concepts through particular examples. In some cases formal definitions are not even provided. For instance, P -values and confidence intervals are important statistical concepts, providing measures relating to the accuracy of a fitted regression line (see Chapter 5). The chapter uses examples, graphs and verbal intuition to demonstrate how they might be used in practice. But no formal definition of a P-value nor derivation of a confidence interval is ever given. This would require the introduction of probability and statistical theory, which is not necessary for using these techniques sensibly in practice. For the reader wishing to learn more about the statistical theory underlying the techniques, many books are available; for instance Introductory Statistics for Business and Economics by Thomas Wonnacott and Ronald Wonnacott (Fourth edition, John Wiley \& Sons, 1990). For those interested in how statistical theory is applied in econometric modeling, Undergraduate Econometrics by R. Carter Hill, William E. Griffiths and George G. Judge (Second edition, John Wiley \& Sons, 2000) provides a useful introduction.
This book reflects my belief that the use of concrete examples is the best way to teach data analysis. Appropriately, each chapter presents several examples as a means of illustrating key concepts. One risk with such a strategy is that some students might
interpret the presence of so many examples to mean that myriad concepts must be mastered before they can ever hope to become adept at the practice of econometrics. This is not the case. At the heart of this book are only a few basic concepts, and they appear repeatedly in a variety of different problems and data sets. The best approach for teaching introductory econometrics, in other words, is to illustrate its specific concepts over and over again in a variety of contexts.

## Organization of the book

In organizing the book, I have attempted to adhere to the general philosophy outlined above. Each chapter covers a topic and includes a general discussion. However, most of the chapter is devoted to empirical examples that illustrate and, in some cases, introduce important concepts. Exercises, which further illustrate these concepts, are included in the text. Data required to work through the empirical examples and exercises can be found in the website which accompanies this book http://www.wileyeurope.com/go/koopdata2ed. By including real-world data, it is hoped that students will not only replicate the examples, but will feel comfortable extending and/or experimenting with the data in a variety of ways. Exposure to realworld data sets is essential if students are to master the conceptual material and apply the techniques covered in this book.
The empirical examples in this book are designed for use in conjunction with the computer package Excel. The website associated with this book contains Excel files. Excel is a simple and common software package. It is also one that students are likely to use in their economic careers. However, the data can be analyzed using many other computer software packages, not just Excel. Many of these packages recognize Excel files and the data sets can be imported directly into them. Alternatively, the website also contains all of the data files in ASCII text form. Appendix B at the end of the book provides more detail about the data.
Mathematical material has been kept to a minimum throughout this book. In some cases, a little bit of mathematics will provide additional intuition. For students familiar with mathematical techniques, appendices have been included at the end of some chapters. However, students can choose to omit this material without any detriment to their understanding of the basic concepts.
The content of the book breaks logically into two parts. Chapters $1-7$ cover all the basic material relating to graphing, correlation and regression. A very short course would cover only this material. Chapters $8-12$ emphasize time series topics and analyze some of the more sophisticated econometric models in use today. The focus on the underlying intuition behind regression means that this material should be easily accessible to students. Nevertheless, students will likely find that these latter chapters are more difficult than Chapters $1-7$.

## Useful background

As mentioned, this book assumes very little mathematical background beyond the pre-university level. Of particular relevance are:

1. Knowledge of simple equations. For instance, the equation of a straight line is used repeatedly in this book.
2. Knowledge of simple graphical techniques. For instance, this book is full of graphs that plot one variable against another (i.e. standard $X Y$-graphs).
3. Familiarity with the summation operator is useful occasionally.
4. In a few cases, logarithms are used.

For the reader unfamiliar with these topics, the appendix at the end of this chapter provides a short introduction. In addition, these topics are discussed elsewhere, in many introductory mathematical textbooks.
This book also has a large computer component, and much of the computer material is explained in the text. There are myriad computer packages that could be used to implement the procedures described in this book. In the places where I talk directly about computer programs, I will use the language of spreadsheets and, particularly, that most common of spreadsheets, Excel. I do this largely because the average student is more likely to have knowledge of and access to a spreadsheet rather than a specialized statistics or econometrics package such as E-views, Stata or MicroFit. ${ }^{1}$ I assume that the student knows the basics of Excel (or whatever computer software package he/she is using). In other words, students should understand the basics of spreadsheet terminology, be able to open data sets, cut, copy and paste data, etc. If this material is unfamiliar to the student, simple instructions can be found in Excel's on-line documentation. For computer novices (and those who simply want to learn more about the computing side of data analysis) Computing Skills for Economists by Guy Judge (John Wiley \& Sons, 2000) is an excellent place to start.

## Appendix 1.1: Mathematical concepts used in this book

This book uses very little mathematics, relying instead on intuition and graphs to develop an understanding of key concepts (including understanding how to interpret the numbers produced by computer programs such as Excel). For most students, previous study of mathematics at the pre-university level should give you all the background knowledge you need. However, here is a list of the concepts used in this book along with a brief description of each.

## The equation of a straight line

Economists are often interested in the relationship between two (or more) variables. Examples of variables include house prices, gross domestic product (GDP), interest rates, etc. In our context a variable is something the economist is interested in and can collect data on. I use capital letters (e.g. $Y$ or $X$ ) to denote variables. A very general way of denoting a relationship is through the concept of a function. A common mathematical notation for a function of $X$ is $\mathrm{f}(X)$. So, for instance, if the economist is interested in the factors that explain why some houses are worth more than others, he/she may think that the price of a house depends on the size of the house. In mathematical terms, he/she would then let $Y$ denote the variable "price of the house" and $X$ denote the variable "size of the house" and the fact that $Y$ depends on $X$ is written using the notation:

$$
Y=\mathrm{f}(X)
$$

This notation should be read " $Y$ is a function of $X$ " and captures the idea that the value for $Y$ depends on the value of $X$. There are many functions that one could use, but in this book I will usually focus on linear functions. Hence, I will not use this general " $\mathrm{f}(X)$ " notation in this book.
The equation of a straight line (what was called a "linear function" above) is used throughout this book. Any straight line can be written in terms of an equation:

$$
Y=\alpha+\beta X
$$

where $\alpha$ and $\beta$ are coefficients, which determine a particular line. So, for instance, setting $\alpha=1$ and $\beta=2$ defines one particular line while $\alpha=4$ and $\beta=-5$ defines a different line.

It is probably easiest to understand straight lines by using a graph (and it might be worthwhile for you to sketch one at this stage). In terms of an $X Y$ graph (i.e. one which measures $Y$ on the vertical axis and $X$ on the horizontal axis) any line can be defined by its intercept and slope. In terms of the equation of a straight line, $\alpha$ is the intercept and $\beta$ the slope. The intercept is the value of $Y$ when $X=0$ (i.e. point at which the line cuts the $Y$-axis). The slope is a measure of how much $Y$ changes when $X$ is changed. Formally, it is the amount $Y$ changes when $X$ changes by one unit. For the student with a knowledge of calculus, the slope is the first derivative, $\frac{d Y}{d X}$.

## Summation notation

At several points in this book, subscripts are used to denote different observations from a variable. For instance, a labor economist might be interested in the wage of every one of 100 people in a certain industry. If the economist uses $Y$ to denote this variable, then he/she will have a value of $Y$ for the first individual, a value of $Y$ for the second individual, etc. A compact notation for this is to use subscripts so that $Y_{1}$
is the wage of the first individual, $Y_{2}$ the wage of the second individual, etc. In some contexts, it is useful to speak of a generic individual and refer to this individual as the $i$-th. We can then write, $Y_{i}$ for $i=1, \ldots, 100$ to denote the set of wages for all individuals.
With the subscript notation established, summation notation can now be introduced. In many cases we want to add up observations (e.g. when calculating an average you add up all the observations and divide by the number of observations). The Greek symbol, $\boldsymbol{\Sigma}$, is the summation (or "adding up") operator and superscripts and subscripts on $\Sigma$ indicate the observations that are being added up. So, for instance,

$$
\sum_{i=1}^{100} Y_{i}=Y_{1}+Y_{2}+\ldots+Y_{100}
$$

adds up the wages for all of the 100 individuals. As other examples,

## $\sum_{i=1}^{3} Y_{i}$

adds up the wages for the first 3 individuals and

$$
\sum_{i=47}^{48} Y_{i}
$$

adds up the wages for the 47th and 48th individuals.
Sometimes, where it is obvious from the context (usually when summing over all individuals), the subscript and superscript will be dropped and I will simply write:

## $\sum Y_{i}$.

## Logarithms

For various reasons (which are explained later on), in some cases the researcher does not work directly with a variable but with a transformed version of this variable. Many such transformations are straightforward. For instance, in comparing the incomes of different countries the variable GDP per capita is used. This is a transformed version of the variable GDP. It is obtained by dividing GDP by population.
One particularly common transformation is the logarithmic one. The logarithm (to the base $B$ ) of a number, $A$, is the power to which $B$ must be raised to give $A$. The notation for this is: $\log _{b}(A)$. So, for instance, if $B=10$ and $A=100$ then the logarithm is 2 and we write $\log _{10}(100)=2$. This follows since $10^{2}=100$. In economics, it is common to work with the so-called natural logarithm which has $B=e$ where $e \approx$ 2.71828. We will not explain where $e$ comes from or why this rather unusual-looking base is chosen. The natural $\log$ arithm operator is denoted by $\ln ;$ i.e. $\ln (A)=\log _{d}(A)$.
In this book, you do not really have to understand the material in the previous paragraph. The key thing to note is that the natural logarithmic operator is a common one (for reasons explained later on) and it is denoted by $\ln (A)$. In practice, it can be easily calculated in a spreadsheet such as Excel (or on a calculator).

## Endnote

1. I expect that most readers of this book will have access to Excel (or a similar spreadsheet or statistics software package) through their university computing labs or on their home computers (note, however, that some of the methods in this book require the Excel Analy sis ToolPak add-in which is not included in some basic installations of Microsoft Works) However, computer software can be expensive and, for the student who does not have access to Excel and is financially constrained, there is an increasing number of free statis tics packages designed using open source software. R. Zelig, which is available at http://gking.harvard.edu/zelig/, is a good example of such a package.

ANALYSIS OF ECONOMIC DATA

by Gary Koop

## Chapter 3: Correlation

Correlation measures numerically the relationship between two variables $X$ and $Y$ (e.g. population density and deforestation).

Correlation between $X$ and $Y$ is symbolised by $r$ or $\mathbf{r}_{\mathrm{XY}}$.

$$
r=\frac{\sum\left(\mathbf{Y}_{\mathbf{i}}-\overline{\mathbf{Y}}\right)\left(\mathbf{X}_{\mathbf{i}}-\overline{\mathbf{X}}\right)}{\sqrt{\sum\left(\mathbf{Y}_{\mathbf{i}}-\overline{\mathbf{Y}}\right)^{2}} \sqrt{\sum\left(\mathbf{X}_{\mathbf{i}}-\overline{\mathbf{X}}\right)^{2}}}
$$

## Understanding Correlation

Example: $\mathrm{Y}=$ deforestation, $\mathrm{X}=$ population density we obtain r = .66.

## What does the fact $r$ is positive mean?

- There is a positive relationship between population density and deforestation.
- Countries with high population densities also tend to have high deforestation rates.
- Countries with low population densities tend to have low deforestation rates.
- Deforestation and population densities both vary across countries. The high/low variation in deforestation rates tends to match up with the high/low variation in population densities.
- $r$ lies between $\mathbf{- 1}$ and +1 .
- Positive values of $r$ indicate positive correlation between $X$ and $Y$, negative values indicate negative correlation, $r=0$ implies $X$ and $Y$ are uncorrelated.
- Larger positive values of $r$ indicate stronger positive correlation. $r=1$ indicates perfect positive correlation.
- More negative values of $r$ indicate stronger negative correlation. $r=-1$ indicates perfect negative correlation.
- The correlation between $Y$ and $X$ is the same as the correlation between $X$ and $Y$.
- The correlation between any variable and itself is $\mathbf{1}$.
- Correlation only measures linear relations


## Understanding Correlation (cont.)

## What does the magnitude of $r$ mean?

$r^{2}$ measures the proportion of the variance in deforestation that matches up with (or is explained by) the variance in population density.

$$
\mathrm{r}^{2}=.66^{2}=.44
$$

44\% of the cross-country variation in deforestation can be explained by the crosscountry variation in population density.

## Why are Variables Correlated?

## Correlation does not necessarily imply causality.

## Example:

The correlation between workers' education levels and wages is strongly positive.

Does this mean education "causes" higher wages? We can't know for sure.

- Possibility 1: Education improves skills and more skilled workers get better paying jobs.

Education causes wages to increase.

- Possibility 2: Individuals are born with quality $A$ which is relevant for success in education and on the job (e.g. intelligence or talent or determination, etc.).

Quality A (NOT education) causes wages to increase.

Why are Variables Correlated? (cont.)

## Example:

Data on N=546 houses sold in Windsor, Canada

Y = sales price of a house
$X$ = the size of the lot the house is on
$r_{X Y}=.54$

- Houses with large lots tend to be worth more than houses with small lots.
- Economic theory tells us that the price of a house should depend on its characteristics.
- Economic theory suggests $\mathbf{X}$ is causing $Y$.
- Here economic theory suggests that correlation does imply causality.


## Why are Variables Correlated? (cont.)

## Example:

Assume:

- Cigarette smoking causes cancer.
- Drinking alcohol does not cause cancer.
- Smokers tend to drink more alcohol.

Suppose we collected data on many elderly people on how much they smoked (X), whether they had cancer $(\mathbf{Y})$ and how much they drank $(\mathrm{Z})$. What correlations would we find?
$\mathbf{r}_{\mathrm{XY}}>0$ Direct causality.
$\mathrm{r}_{\mathrm{YZ}}>0$ This is does NOT reflect causality.

## Why are Variables Correlated? (cont.)

## Example:

High rural population density (X) causes farmers to clear new land in forested areas (Z) which in turn causes deforestation ( Y ).

Here we would find $\mathrm{r}_{\mathrm{XY}}>0$ and $\mathrm{r}_{\mathrm{ZY}}>0$.
$X$ (population density) is an indirect (or proximate) cause of $Y$ (deforestation).

Z (agricultural clearance) is a direct (or immediate) cause of $Y$ (deforestation).

Why are Variables Correlated?
Summary

Correlations can be very suggestive, but cannot on their own establish causality.

- Correlation + a sensible theory suggests (but does not prove) causality


## Understanding Correlation through XY-plots

## Figure 3.1: House price versus lot size

Think of drawing a straight line that best fits the points in the XY-plot. It will have positive slope.

Positive slope=positive relationship=positive correlation.

Figure 3.2: XY plot with $\underline{X X Y}_{\underline{X}}=1$
All points fit on a straight upward sloping line.

## Understanding Correlation through XY-plots (continued)

## Figure 3.3: XY plot with $\mathbf{r}_{\underline{X Y}}=.51$

Points still exhibit an upward sloping pattern, but much more scattered.

Figure 3.4: XY plot with $\underline{r}_{\underline{X Y}}=0$

Completely random scattering of points.

Figure 3.5: XY plot with $\mathbf{r}_{\underline{X Y}}=-.51$

Think of drawing a straight line that best fits the points in the XY-plot. It will have negative slope.

Negative slope=negative relationship=negative correlation.

## Correlation Among Several Variables

Correlation relates precisely two variables.
What to do with three or more? Usually use regression (next chapter).

Or you can calculate the correlation between every possible pair of variables.

Example: Three variables: X, Y and Z.

Can calculate three correlations: $\mathbf{r}_{\mathrm{xy}}, \mathrm{r}_{\mathrm{xz}}$ and $\mathbf{r}_{\mathrm{yz}}$.

Four variables: $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ and W .

Can calculate six correlation: $\mathbf{r}_{\mathrm{xy}}, \mathbf{r}_{\mathrm{x}}, \mathbf{r}_{\mathrm{xw}}, \mathbf{r}_{\mathrm{yz}}, \mathbf{r}_{\mathrm{yw}}$ and $\mathrm{r}_{\mathrm{zw}}$.

M variables: $\mathbf{M} \times(\mathbf{M}-1) / 2$ correlations.

## A Correlation Matrix

Example:
Column $1=\mathrm{X}$, Column $2=\mathrm{Y}$, Column $3=\mathrm{Z}$

|  | Column1 | Column2 | Column3 |
| :--- | ---: | ---: | ---: |
| Column1 | 1 |  |  |
| Column2 | 0.3182369 | 1 |  |
| Column3 | -0.1309744 | 0.0969959 | 1 |

$\mathrm{r}_{\mathrm{xy}}=\mathbf{0 . 3 1 8 2 3 6 9}$
$r_{x z}=\mathbf{- 0 . 1 3 0 9 7 4 4}$
$r_{y z}=0.0969959$

# ANALYSIS OF ECONOMIC DATA <br> by Gary Koop 

## Regression as a Best Fitting Line

Example:

See Figure 2.3: XY-plot of deforestation versus population density.

Regression fits a line through the points in the XY-plot that best captures the relationship between deforestation and population density.

Question: What do we mean by "best fitting" line?

## Simple Regression: Theory (cont.)

1. Even if straight line relationship were true, we would never get all points on an XY-plot lying precisely on it due to measurement error.
2. True relationship probably more complicated, straight line may just be an approximation.
3. Important variables which affect $Y$ may be

Due to 1., 2. and 3. we add an error.

[^1] omitted.
$\alpha=0$
$\beta=.8=$ marginal product of labour.

## Simple Regression: Theory

Assume a linear relationship exists between $Y$ and $X$ :

$$
\mathbf{Y}=\alpha+\beta \mathbf{X}
$$

$\alpha=$ intercept of line
$\beta=$ slope of line

Example:
$Y=$ output of a good, $X=$ labour input

$$
\mathbf{Y}=.8 \times \mathbf{X}
$$

The Simple Regression Model

$$
\mathbf{Y}=\alpha+\beta \mathbf{X}+\mathbf{e}
$$

where $\mathbf{e}$ is an error.

What we know: $X$ and $Y$.

What we do not know: $\alpha, \beta$ or e .

Regression analysis uses data ( $X$ and $Y$ ) to make a guess or estimate of what $\alpha$ and $\beta$ are.

Notation: $\hat{\alpha}$ and $\hat{\boldsymbol{\beta}}$ are the estimates of $\alpha$ and $\beta$.

## How do we choose $\hat{\alpha}$ and $\hat{\beta}$ ?

Consider the following reasoning:

1. We want to find a best fitting line through the XY-plot.
2. With more than two points it is not possible to find a line that fits perfectly through all points. (See Figure 4.1)
3. Hence, find "best fitting" line which makes the residuals as small as possible.
4. What do we mean by "as small as possible"? The one that minimizes the sum of squared residuals.
5. Hence, we obtain the "ordinary least squares" or OLS estimator.

## Distinction Between Errors and Residuals

True Regression Line:

$$
\mathbf{Y}=\alpha+\beta \mathbf{X}+\mathbf{e}
$$

$$
\mathbf{e}=\mathbf{Y}-\alpha-\beta \mathbf{X}
$$

e = error

## Estimated Regression Line:

$$
\mathbf{Y}=\hat{\alpha}+\hat{\beta} \mathbf{X}+\mathbf{u}
$$

$$
\mathbf{u}=\mathbf{Y}-\hat{\alpha}-\hat{\beta} \mathbf{X}
$$

$$
\mathbf{u}=\text { residual }
$$

## Derivation of OLS Estimator

- We have data on $\mathrm{i}=1$,.., N individuals which we call $Y_{i}$ and $X_{i}$.
- Any line we fit/choice of $\hat{\alpha}$ and $\hat{\boldsymbol{\beta}}$ will yield residuals $\mathbf{u}_{\mathbf{i}}$.
- Sum of squared residuals $=\operatorname{SSR}=\Sigma \mathrm{u}_{\mathrm{i}}{ }^{2}$.
- OLS estimator chooses $\hat{\alpha}$ and $\hat{\beta}$ to minimise SSR.

Solution:

$$
\hat{\beta}=\frac{\sum\left(Y_{i}-\bar{Y}\right)\left(X_{i}-\bar{X}\right)}{\sum\left(X_{i}-\bar{X}\right)^{2}}
$$

and

$$
\hat{\alpha}=\overline{\mathbf{Y}}-\hat{\boldsymbol{\beta}} \times \bar{X}
$$

## Jargon of Regression

## Regression and Causality

- $Y=$ dependent variable.
- $X=$ explanatory (or independent) variable.
- $\alpha$ and $\beta$ are coefficients.
- $\hat{\alpha}$ and $\hat{\beta}$ are OLS estimates of coefficients
- "Run a regression of $Y$ on $X$ " "How much of the variability in $Y$ can be explained by X?"


## How to decide which is dependent variable

- Ideally, explanatory variable should be the one which causes/influences the dependent variable.
- Ideally, X causes Y.

If you can, build models where causality assumptions make sense

## Examples:

- Increases in $X=$ population density cause $Y=$ deforestation to increase (not vice versa).
- Increasing $X$ = the lot size of a house causes $Y$ $=$ its value to increase (not vice versa).
- Increasing $X=$ advertising expenditures causes $Y=$ company sales to increase (not vice versa).


## Regression and Causality (cont.)

In practice, great care must be taken in interpreting regression results as reflecting causality. Why?

- In some cases, your assumption that $\mathbf{X}$ causes Y may be wrong.
- In some cases, you may not know whether $X$ causes $\mathbf{Y}$.
- In some cases, $X$ may cause $Y$ but $Y$ may also cause $X$ (e.g. exchange rates and interest rates).
- In some cases, the whole concept of causality may be inappropriate.

Formally, the question regression addresses is:

## Interpreting OLS Estimates

$$
\mathbf{Y}=\hat{\alpha}+\hat{\beta} \mathbf{X}+\mathbf{u}
$$

Ignore u for the time being (focus on regression line)

## Interpretation of $\underline{\hat{\alpha}}$

- Estimated value of $\mathbf{Y}$ if $\mathbf{X}=\mathbf{0}$.
- This is often not of interest.


## Example:

$X=$ lot size, $Y=$ house price
$\hat{\alpha}=$ estimated value of a house with lot size $=0$

Interpreting OLS Estimates (cont.)

$$
\mathbf{Y}=\hat{\alpha}+\hat{\boldsymbol{\beta}} \mathbf{X}+\mathbf{u}
$$

## Interpretation of $\underline{\hat{\beta}}$

1. $\hat{\beta}$ is estimate of the marginal effect of $X$ on $Y$
2. Using regression model:

$$
\frac{d Y}{d X}=\hat{\beta}
$$

3. A measure of how much $Y$ tends to change when you change $X$.
4. "If $X$ changes by 1 unit then $Y$ tends to change by $\hat{\boldsymbol{\beta}}$ units", where "units" refers to what the variables are measured in (e.g. \$, £, $\%$, hectares, metres, etc.).

## Example: Deforestation and Population

 Density (continued)Interpretation of $\underline{\hat{\beta}}$
a) "If population density increases by 1 person per 1,000 hectares, then deforestation will tend to increase by $\mathbf{. 0 0 0 8 4 2 \%}$ per year"
b) "If population density increases by 100 people per 1,000 hectares, then deforestation will tend to increase by $\mathbf{. 0 8 4 2 \%}$ per year"

Note: if deforestation increased by $\mathbf{. 0 8 4 2 \%}$ per year an additional $5 \%$ of the forest will be lost over 50 years.

Example: Deforestation and Population Density

Development economists have theories which imply that increasing population density should increase deforestation.

Thus:
$Y=$ deforestation (annual \% change) = dependent variable

X = population density (people per thousand hectares) $=$ explanatory variable

Using data on $\mathbf{N}=\mathbf{7 0}$ tropical countries we find:

$$
\hat{\beta}=0.000842
$$

## Example: Cost of Production in the Electric Utility Industry

Data on $\mathrm{N}=123$ electric utility companies in the U.S.
$Y=$ cost of production (millions of \$)
X = output (thousands of kilowatt hours, Kwh)

$$
\hat{\beta}=.005
$$

"Increasing output by $\mathbf{1 , 0 0 0}$ Kwh tends to increase costs by $\$ \mathbf{5 , 0 0 0 "}$

Note: $.005 \times \mathbf{1 , 0 0 0 , 0 0 0}=\mathbf{5 , 0 0 0}$
"Decreasing output by $\mathbf{1 , 0 0 0}$ Kwh tends to decrease costs by $\mathbf{\$ 5 , 0 0 0 "}$

Example: The Effect of Advertising on Sales

Data on $\mathrm{N}=\mathbf{8 4}$ companies in the U.S.
$\mathbf{Y}=$ sales (millions of $\$$ )
$\mathrm{X}=$ advertising expenditure (millions of \$)

$$
\hat{\beta}=.218
$$

"Increases in advertising of $\$ 1,000,000$ are associated with increases in sales of $\$ \mathbf{2 1 8 , 0 0 0}$."

## Residual Analysis (cont.)

- Good fitting models have small residuals (i.e. small SSR)
- If residual is big for one observation (relative to other observations) then it is an outlier.
- Looking at fitted values and residuals can be very informative.
"How well does the regression line fit through the points on the XY-plot?"
"Are there any outliers which do not fit the general pattern?"


## Concepts:

1. Fitted value of dependent variable:

$$
\hat{\mathbf{Y}}_{\mathrm{i}}=\hat{\alpha}+\hat{\beta} \times X_{i}
$$

- $\mathbf{Y}_{i}$ does not lies on the regression line, $\hat{\mathbf{r}}_{\mathbf{i}}$ does lie on line.
- What would you do if you did not know $\mathbf{Y}_{\mathbf{i}}$, but only $\mathbf{X}_{i}$ and wanted to predict what $\mathbf{Y}_{i}$ was? Answer $\hat{\mathbf{Y}}_{\mathrm{i}}$

2. Residual, $u_{i}$, is: $\mathbf{Y}_{\mathbf{i}}-\hat{\mathbf{Y}}_{\mathbf{i}}$

## $\mathbf{R}^{2}$ : A Measure of Fit

Intuition:
"Variability" = (e.g.) how deforestation rates vary across countries

Total variability in dependent variable $\mathbf{Y}=$
Variability explained by the explanatory variable ( $X$ ) in the regression

Variability that cannot be explained and is left as an error.

## $\mathbf{R}^{\mathbf{2}}$ : A Measure of Fit (cont.)

## $\mathbf{R}^{\mathbf{2}}$ : A Measure of Fit (cont.)

In mathematical terms,
TSS = RSS + SSR
where TSS = Total sum of squares

$$
\mathrm{TSS}=\sum\left(\mathrm{Y}_{\mathrm{i}}-\overline{\boldsymbol{Y}}\right)^{\mathbf{2}}
$$

Note similarity to formula for variance.

RSS = Regression sum of squares

$$
\mathbf{R S S}=\Sigma\left(\hat{\mathbf{Y}}_{\mathbf{i}}-\overline{\mathbf{Y}}\right)^{\mathbf{2}}
$$

SSR = sum of squared residuals

## Properties of $\mathbf{R}^{\mathbf{2}}$

- $0 \leq \mathrm{R}^{2} \leq 1$
- $R^{2}=1$ means perfect fit. All data points exactly on regression line (i.e. $S S R=0$ ).
- $R^{2}=0$ means $X$ does not have any explanatory power for $Y$ whatsoever (i.e. $X$ has no influence on $\mathbf{Y}$ ).
- Bigger values of $\mathbf{R}^{2}$ imply $X$ has more explanatory power for $Y$.
- $R^{2}$ is equal to the correlation between $X$ and $\mathbf{Y}$ squared (i.e. $\mathbf{R}^{2}=\mathbf{r}_{\mathrm{xy}}{ }^{2}$ )

$$
\mathrm{R}^{2}=1-\frac{S S R}{T S S}
$$

or (equivalently):

$$
\mathrm{R}^{2}=\frac{R S S}{T S S}
$$

$R^{2}$ is a measure of fit (i.e. how well does the regression line fit the data points)

## Properties of $\mathbf{R}^{\mathbf{2}}$ (cont.)

$\mathbf{R}^{\mathbf{2}}$ measures the proportion of the variability in Y that can be explained by X .

## Example:

In regression of $\mathbf{Y}=$ deforestation on $\mathbf{X}=$ population density we obtain $R^{2}=0.44$
" $44 \%$ of the cross-country variation in deforestation rates can be explained by the cross-country variation in population density"

## Nonlinearity

So far regression of $\mathbf{Y}$ on $\mathbf{X}$ :

$$
\mathbf{Y}=\alpha+\beta \mathbf{X}+\mathbf{e}
$$

Could do regression of $Y\left(\operatorname{or} \ln (Y)\right.$ or $\left.Y^{2}\right)$ on $X^{2}$ (or $1 / X$ or $\ln (X)$ or $X^{3}$, etc.) and same techniques and results would hold.
E.g.

$$
\mathrm{Y}=\alpha+\beta \mathrm{X}^{2}+e
$$

How might you know if relationship is nonlinear?

Answer: Careful examination of XY-plots or residual plots.

Figures 4.2, 4.3 and 4.4.

## Chapter 5: Statistical Aspects of Regression

## What Factors Affect Accuracy of OLS Estimates?

## Graphical Intuition:

- Figure 5.1 (small number of data points)
- Figure 5.2 (large number of data points but very scattered)
- Figure 5.3 (large number of data points but not very scattered)
- Figure 5.4 (large number of data points, but clustered near one value for $\mathbf{X}$ )


## What Factors Affect Accuracy of OLS Estimates?

Consider fitting a line through the XY-plots in Figures 5.1-5.4.

You would be most confident in the line you fit in Figure 5.3

## Larger number of data points + less scattering (i.e. less variability in errors) + more variability in

 $X=$ more accurate estimates.Note: Figures 5.1, 5.2, 5.3 and 5.4 all contain artificially generated data with $\alpha=0, \beta=1$.

## A Confidence Interval for $\boldsymbol{\beta}$

- Uncertainty about accuracy of the estimate $\hat{\beta}$ can be summarised in a "confidence interval"
- $\mathbf{9 5 \%}$ confidence interval for $\beta$ is given by:
- $t_{b}$ is a "critical value" from the "Student $t$ distribution" --- calculated automatically in Excel
- $s_{b}=$ standard error of $\hat{\boldsymbol{\beta}}$ is a measure of the accuracy of $\hat{\boldsymbol{\beta}}$

$$
\stackrel{s}{\mathrm{~b}}=\sqrt{\frac{\operatorname{SSR}}{(N-2) \times \sum\left(X_{i}-\bar{X}\right)^{2}}}
$$

## A Confidence Interval for $\hat{\boldsymbol{\beta}}$ (cont.)

- $t_{b}$ controls the confidence level (e.g. $t_{b}$ is bigger for $\mathbf{9 5 \%}$ confidence than $\mathbf{9 0 \%}$ ).
- $s_{b}$ varies directly with SSR (i.e. how variable the residuals are)
- $s_{b}$ varies inversely with $N$, the number of data points
- $\mathbf{s}_{\mathbf{b}}$ varies inversely with $\Sigma\left(\mathrm{X}_{\mathbf{i}}-\bar{X}\right)^{2}$, which is related to the variance/variability of $X$.

Note: Excel calculates the confidence interval for you and labels bounds of confidence interval as "Lower $95 \%$ " and "Upper $95 \%$ "

## Example: Confidence Intervals for the Data sets in Figures 5.1-5.4

| Data Set | $\hat{\boldsymbol{\beta}}$ | 90\% <br> Confid. <br> Interval | 95\% <br> Confid. <br> Interval | 99\% <br> Confid. <br> Interval |
| :--- | :---: | :--- | :--- | :--- |
| Figure 5.1 | .91 | $[-.92,2.75]$ | $[-1.57,3.39]$ | $[-3.64,5.47]$ |
| Figure 5.2 | 1.04 | $[.75,1.32]$ | $[.70,1.38]$ | $[.59,1.49]$ |
| Figure 5.3 | 1.00 | $[.99,1.01]$ | $[.99,1.02]$ | $[.98,1.03]$ |
| Figure 5.4 | 1.52 | $[-1.33,4.36]$ | $[-1.88,4.91]$ | $[-2.98,6.02]$ |

## Intuition of Confidence Interval

- Useful (but formally incorrect) intuition: "There is a $95 \%$ probability that the true value of $\beta$ lies in the confidence interval".
- Correct intuition: "If you repeatedly use the above formula for calculating a confidence interval, $95 \%$ of the intervals you construct will contain the true value for $\beta$ ".
- Can choose any level of confidence you want (e.g. 90\%, 99\%).
$\mathbf{Y}=$ deforestation
X = population density

$$
\hat{\beta}=.000842
$$

95\% Confidence interval: [.00061,.001075]

## Example: The Regression of Lot Size on House Price

OLS results:

$$
Y=34,136+6.59 X
$$

. The OLS estimate of the marginal effect of $X$ on $Y$ is 6.59.
. "Increasing lot size by an extra square foot is associated with a $\$ 6.59$ increase in house price."
. The $95 \%$ confidence interval for $\beta$ is [5.72,7.47].
. "We are $95 \%$ confident that the effect of lot size on house is at least $\$ 5.72$ and at most \$7.47."

## Hypothesis Testing (cont.)

Q: What do we mean by "big" and "small"?

A: Look at P-value.

- If P-value $\leq .05$ then $t$ is "big" and conclude $\beta \neq 0$.
- If P-value >. 05 then $t$ is "small" and conclude $\beta=0$.
- Useful (but formally incorrect) intuition: P -value measures the probability that $\beta=0$.
- $.05=5 \%=$ level of significance
- Other levels of significance (e.g. 1\% or 10\%) occasionally used


## Hypothesis Testing

- Test whether $\beta=0$ (i.e. whether $\mathbf{X}$ has any explanatory power)
- One way of doing it: look at confidence interval, check whether it contains zero. If no, then you are confident $\beta \neq 0$.
- Alternative (equivalent) way is to use " $t$ statistic" (often called "t-ratio")

- "Big" values for $\mathbf{t}$ indicate $\beta \neq 0$.
- "Small" values for $\mathbf{t}$ indicate it $\beta=0$.
(more concretely: $\beta$ might be 0 )

95\% Confidence interval: [.00061,.001075]

Confidence interval does not contain zero, so conclude that $\beta \neq \mathbf{0}$.

Alternatively:
t-ratio is 7.227937. Is this big?

Yes, the $P$-value is $5.5 \times 10^{-10}$ which is much less than . 05.

Hence, we conclude again that $\beta \neq 0$.

- "The coefficient on population density is significantly different from zero."
- "Population density has statistically significant explanatory power for deforestation."
- "The hypothesis that $\beta=0$ can be rejected at the $5 \%$ significance level."
- Test whether $R^{2}=0$ (i.e. whether $X$ has any explanatory power)
- Note: In simple regression testing $R^{2}=0$ and $\beta=0$ are the same, but in multiple regression they will be different.
- F-statistic is a test statistic analogous to tstatistic (e.g. small values of it indicate $R^{2}=0$ ).

$$
F=\frac{(N-2) R^{2}}{\left(1-R^{2}\right)}
$$

## Testing on $\mathbf{R}^{\mathbf{2}}$ : The F-statistic (cont.)

- For test with 5\% level of significance:
- If $\mathbf{P}$-value is > . 05 conclude $\mathbf{R}^{2}=\mathbf{0}$.
- If $\mathbf{P}$-value is $\leq .05$ conclude $R^{2} \neq 0$.
- $\mathbf{P}$-value $=$ Significance $\mathbf{F}=5.5 \times 10^{-10}$.
- Since $\mathbf{P}$-value $<.05$ conclude $\mathbf{R}^{2} \neq 0$.
- Population density does have explanatory power for $Y$.
- Excel calls the P-value for this test "Significance F"


## Example: The Regression of Deforestation on Population Density (cont.)

Data on N=546 houses sold in Windsor, Canada

Dependent variable:
$Y=$ sales price of house

Explanatory variables:
$X_{1}=$ lot size of property (in square feet)
$X_{2}=$ number of bedrooms
$X_{3}=$ number of bathrooms
$\mathrm{X}_{4}=$ number of storeys (excluding basement)

## OLS Estimation

## Statistical Aspects of Multiple Regression

- Multiple regression model:

$$
Y_{i}=\alpha+\beta_{1} X_{1 i}+\ldots+\beta_{k} X_{k i}+e_{i}
$$

- OLS estimates: $\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}}_{1}, \hat{\boldsymbol{\beta}}_{2}, \ldots, \hat{\boldsymbol{\beta}}_{k}$
- Minimise Sum of Squared Residuals:

$$
\operatorname{SSR}=\sum\left(\mathrm{Y}_{\mathrm{i}}-\hat{\alpha}-\hat{\beta}_{1} X_{1 i}-\ldots-\hat{\beta}_{k} X_{k i}\right)^{2}
$$

- Solution to minimisation problem: A mess
- Excel will calculate OLS estimates
- $\mathbf{R}^{2}$ still a measure of fit with same interpretation (although now it is no longer simply the correlation between $Y$ and $X$ squared).

Statistical Aspects of Multiple Regression (cont.)

- Can test $\mathbf{R}^{2}=0$ in same manner as for simple regression.
- If you find $\mathbf{R}^{2} \neq 0$ then you conclude that the explanatory variables together provide significant explanatory power (Note: this does not necessarily mean each individual explanatory variable is significant).
- Confidence intervals can be calculated for each individual coefficient as before.
- Can test $\beta_{\mathrm{j}}=\mathbf{0}$ for each individual coefficient ( $\mathrm{j}=1,2, . ., \mathrm{k}$ ) as before.


## Verbal intuition

- $\beta_{j}$ is the marginal effect of $X_{j}$ on $Y$, ceteris paribus
- $\beta_{\mathrm{j}}$ is the effect of a small change in the j th explanatory variable on the dependent variable, holding all the other explanatory variables constant.

Interpreting OLS Estimates in Multiple
Regression Model

## Mathematical Intuition

- Total vs. partial derivative
- Simple regression: $\frac{\mathbf{d Y}}{\mathbf{d X}}=\beta$
- Multiple Regression: $\frac{\partial \mathbf{Y}}{\partial \mathbf{X}} \mathbf{j}_{\mathbf{j}}=\beta_{j}$


## Interpreting OLS Estimates in Multiple Regression Model (cont.)

## Example: Explaining House Prices (cont.) <br> ```(cont.)```

|  | Coeff. | St.Err | t-Stat | P-val. | Lower <br> $\mathbf{9 5 \%}$ | Upper <br> $\mathbf{9 5 \%}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Interc. | $-\mathbf{4 0 1 0}$ | $\mathbf{3 6 0 3}$ | $\mathbf{- 1 . 1 1 3}$ | $\mathbf{0 . 2 6 6}$ | $\mathbf{- 1 1 0 8 7}$ | $\mathbf{3 0 6 8}$ |
| Size | $\mathbf{5 . 4 2 9}$ | $\mathbf{0 . 3 6 9}$ | $\mathbf{1 4 . 7 0 3}$ | 2.E-41 | 4.704 | $\mathbf{6 . 1 5 5}$ |
| Bed. | $\mathbf{2 8 2 5}$ | $\mathbf{1 2 1 5}$ | 2.325 | $\mathbf{0 . 0 2 0}$ | $\mathbf{4 3 8 . 3}$ | $\mathbf{5 2 1 1}$ |
| Bath. | $\mathbf{1 7 1 0 5}$ | $\mathbf{1 7 3 4}$ | $\mathbf{9 . 8 6 2}$ | 3.E-21 | $\mathbf{1 3 6 9 8}$ | $\mathbf{2 0 5 1 2}$ |
| Storeys | 7635 | $\mathbf{1 0 0 8}$ | 7.574 | $\mathbf{1 . E - 1 3}$ | 5655 | $\mathbf{9 6 1 5}$ |

- $\mathbf{R}^{2}=.54$ and the $P$-value for testing $\mathbf{R}^{2}=0$ (which is labelled "Significance $F$ " by Excel) is 1.18E-88.
- Fitted regression line:

$$
\begin{gathered}
\hat{Y}=-4010+5.429 X_{1}+2825 X_{2}+17105 X_{3}+ \\
7635 X_{4}
\end{gathered}
$$

Example: Explaining House Prices<br>(cont.)

Example: Explaining House Prices<br>(cont.)

Since $\hat{\beta}_{1}=5.43:$

- An extra square foot of lot size will tend to add $\$ 5.43$ onto the price of a house, ceteris paribus.
- For houses with the same number of bedrooms, bathrooms and storeys, an extra square foot of lots size will tend to add $\$ 5.43$ onto the price of a house.
- If we compare houses with the same number of bedrooms, bathrooms and storeys, those with larger lots tend to be worth more. In particular, an extra square foot in lot size is associated with an increased price of $\$ 5.43$.


## Pitfalls of Using Simple Regression in a Multiple Regression Context

- In multiple regression above, coefficient on number of bedrooms was $\mathbf{2 , 8 2 4 . 6 1}$.
- A simple regression of $Y=$ house price on $X=$ number of bedrooms yields a coefficient estimate of $13,269.98$.
- Why are these two coefficients on the same explanatory variable so different? i.e. 13,269.98>>>2,824.61.

Answer 1: They just come from two different regressions which control for different explanatory variables (different ceteris paribus conditions).

Since $\hat{\boldsymbol{\beta}}_{2}=$ 2,824.61:

- Adding one bedroom to your house will tend to increase its value by $\mathbf{\$ 2 , 8 2 4 . 6 1}$, ceteris paribus.
- If we consider houses with comparable lot sizes and numbers of bathrooms and storeys, then those with an extra bedroom tend to be worth \$2,824.61 more.


## Pitfalls of Using Simple Regression in a Multiple Regression Context (cont.)

Answer 2:

- Imagine a friend asked: "I have 2 bedrooms and I am thinking of building a third, how much will it raise the price of my house?"
- Simple regression: "Houses with 3 bedrooms tend to cost $\$ 13,269.98$ more than houses with 2 bedrooms"
- Does this mean adding a $3^{\text {rd }}$ bedroom will tend to raise price of house by $\$ 13,269.98$ ? Not necessarily, other factors influence house prices.
- Houses with three bedrooms also tend to be desirable in other ways (e.g. bigger, with larger lots, more bathrooms, more storeys, etc.). Call these "good houses".
- Simple regression notes "good houses" tend to be worth more than others.


## Pitfalls of Using Simple Regression in a Multiple Regression Context (cont.)

## Pitfalls of Using Simple Regression in a Multiple Regression Context (cont.)

Statistical evidence:
Correlation matrix:

|  | Sale Price | Lot size | \#bed | \#bath | \#storey |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Sale | 1 |  |  |  |  |
| price |  |  |  |  |  |
| Lot size | 0.5358 | 1 |  |  |  |
| \#bed | 0.3664 | 0.1519 | 1 |  |  |
| \#bath | 0.5167 | 0.1938 | 0.3738 | 1 |  |
| \#storeys | 0.4212 | 0.0837 | 0.4080 | 0.3241 | 1 |

- Tell your friend: "Adding a third bedroom will tend to raise your house price by \$2,824.61".
- Multiple regressions which contains all (or most) of house characteristics will tend to be more reliable than simple regression which only uses one characteristic.


## Omitted Variable Bias

"Omitted variable bias" is a statistical term for these issues.

## IF

1. We exclude explanatory variables that should be present in the regression,

## AND

2. these omitted variables are correlated with the included explanatory variables,

## THEN

3. the OLS estimates of the coefficients on the included explanatory variables will be biased.

## Omitted Variable Bias (cont.)

## Example:

- Simple regression used $Y=$ house prices and $X=$ number of bedrooms.
- Many important determinants of house prices omitted.
- Omitted variables were correlated with number of bedrooms. Hence, the OLS estimate from the simple regression $\hat{\beta}=13,269.98$ was biased.


# Practical Advice for Selecting Explanatory Variables 

- Include (insofar as possible) all explanatory variables which you think might possibly explain your dependent variable. This will reduce the risk of omitted variable bias.
- However, including irrelevant explanatory variables reduces accuracy of estimation and increases confidence intervals. So do t-tests to decide whether variables are significant. Run a new regression omitting the explanatory variables which are not significant.


## Multicollinearity (cont.)

## Example:

$$
Y=\text { exchange rate }
$$

Explanatory variable(s) = interest rate
$\mathrm{X}_{1}=$ bank prime rate
$X_{2}=$ Treasury bill rate

Using both $X_{1}$ and $X_{2}$ will probably cause multicollinearity problem

Solution: Include either $\mathbf{X}_{1}$ or $\mathbf{X}_{\mathbf{2}}$ but not both.

In some cases this "solution" will be unsatisfactory if it causes you to drop out explanatory variables which economic theory says should be there.

- Intuition: If explanatory variables are highly correlated with one another then regression model has trouble telling which individual variable is explaining $Y$.
- Symptom: Individual coefficients may look insignificant, but regression as a whole may look significant (e.g. $\mathbf{R}^{2}$ big, F-stat big).
- Looking at a correlation matrix for explanatory variables can often be helpful in revealing extent and source of multicollinearity problem.


## Example: Multicollinearity Illustrated using Artificial Data

True Model:

$$
Y=.5 X_{1}+2 X_{2}+e
$$

Correlation between $X_{1}$ and $X_{2}=.98$

|  | Coeff. | St. <br> Error | t-Stat | P-val. | Lower <br> $\mathbf{9 5 \%}$ | Upper <br> $\mathbf{9 5 \%}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Inter. | $\mathbf{. 1 6 6 2}$ | $\mathbf{. 1 0 2 5}$ | $\mathbf{1 . 5 7 9}$ | $\mathbf{. 1 2 1 1}$ | -.0456 | $\mathbf{. 3 7 8 0}$ |
| X1 | 2.084 | $\mathbf{. 9 5 2 9}$ | 2.187 | .0338 | $\mathbf{. 1 6 6 7}$ | $\mathbf{4 . 0 0 1}$ |
| X2 | $\mathbf{. 1 4 7 8}$ | . $\mathbf{. 9 6 5 8}$ | $\mathbf{. 1 5 3 0}$ | $\mathbf{. 8 7 9 0}$ | $\mathbf{- 1 . 7 9 5}$ | $\mathbf{2 . 0 9 1}$ |

- $\mathbf{R}^{2}=.76$
- $P$-value for testing $\mathrm{R}^{2}=\mathbf{0}$ is $1.87 \mathrm{E}-15$.
- We want coefficient estimates of roughly . 5 and 2 - this is not what we are getting. Plus X 2 is not significant.


## Example: Multicollinearity Illustrated using Artificial Data (cont.)

Drop $X_{2}$ and re-run the regression:

|  | Coeff. | St. <br> Error | t-Stat | P-val. | Lower <br> $95 \%$ | Upper <br> $95 \%$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Inter. | $\mathbf{. 1 6 6 7}$ | $\mathbf{. 1 0 4 1}$ | 1.601 | $\mathbf{. 1 1 6 0}$ | -.0427 | .3761 |
| X1 | 2.227 | $\mathbf{. 1 7 8 8}$ | $\mathbf{1 2 . 4 5 4}$ | 1.E-16 | 1.867 | 2.586 |

- $\mathrm{R}^{2}=.76$
- $P$-value for testing $R^{2}=0$ is $1.2 E-16$.
- Coefficient on X1 is significant, but is nowhere near .5!


## Chapter 7: Regression with Dummy Variables

## Analysis of Variance

- Analysis of Variance $=$ ANOVA
- Dummy variable is either 0 or 1.
- Use to turn qualitative (Yes/No) data into 1/0.


## Example: Explaining House Prices

Dependent variable: Price of house
Explanatory variables:

- $D_{1}=1$ if the house has a driveway (=0 if it does not).
- $D_{2}=1$ if the house has a recreation room (=0 if not).
- $D_{3}=1$ if the house has a basement $(=0$ if not).
- $\mathrm{D}_{4}=1$ if the house has gas central heating (=0 if not).
- $D_{5}=1$ if the house has air conditioning (=0 if not).


## Simple Regression with a Dummy <br> Variable

$$
\mathbf{Y}=\alpha+\beta \mathbf{D}+\mathbf{e}
$$

- OLS estimation, confidence intervals, testing, etc. carried out in standard way.
- Interpretation a little different.
- Statistical technique commonly used in many fields (but not much used in economics).
- ANOVA is just a special case of regression with dummy variables.
- Regression with dummy variables is much more powerful and general tool.


## Simple Regression with a Dummy <br> Variable

- Fitted value for $\mathbf{i}^{\text {th }}$ observation (point on regression line):

$$
\hat{\mathbf{Y}}_{\mathbf{i}}=\hat{\alpha}+\hat{\beta} D_{i}
$$

- Since $D_{i}=0$ or 1 either

$$
\hat{\mathbf{Y}}_{i}=\hat{\alpha}
$$

or

$$
\hat{\mathbf{Y}}_{\mathbf{i}}=\hat{\alpha}+\hat{\beta}
$$

## Example: Explaining house prices

(continued)

- Regress $Y$ = house price on $D=$ dummy for air conditioning ( $=1$ if house has air conditioning, = 0 otherwise).
- Result:

$$
\begin{gathered}
\hat{\alpha}=59,885 \\
\hat{\beta}=25,996 \\
\hat{\alpha}+\hat{\beta}=85,881
\end{gathered}
$$

- Average price of house with air conditioning is \$85,881
- Average price of house without air conditioning is $\$ 59,885$

|  | Coeff. | St. <br> Error | t Stat | P- <br> value | Lower <br> $95 \%$ | Upper <br> $95 \%$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Inter. | 47099.1 | 2837.6 | 16.60 | 2.E-50 | 41525 | 52673 |
| D1 | 21159.9 | 3062.4 | 6.91 | 1.E-11 | 15144 | 27176 |
| D2 | 16023.7 | 2788.6 | 5.75 | 1.E-08 | 10546 | 21502 |

1. If $D_{1}=1$ and $D_{2}=1$, then
$\hat{Y}=\hat{\alpha}+\hat{\beta}_{1}+\hat{\beta}_{2}=47,099+21,160+16,024=84,283$
"The average price of houses with a driveway and rec room is $\$ 84,283$ ".

## Multiple Regression with Dummy Variables

$$
Y=\alpha+\beta_{1} D_{1}+\ldots+\beta_{k} D_{k}+e
$$

## Example: Explaining house prices (continued)

Regress $Y=$ house price on $D_{1}=$ driveway dummy and $D_{2}=$ rec room dummy

Four types of houses:

1. Houses with a driveway and a rec room ( $D_{1}=1$, $\mathrm{D}_{2}=1$ )
2. Houses with a driveway but no rec room ( $\mathrm{D}_{1}=1, \mathrm{D}_{2}=0$ )
3. Houses with a rec room but no driveway ( $\mathrm{D}_{1}=0, \mathrm{D}_{2}=1$ )
4. Houses with no driveway and no rec room ( $\mathrm{D}_{1}=0, \mathrm{D}_{2}=0$ )

## Example: Explaining house prices

(continued)
2. If $D_{1}=1$ and $D_{2}=0$, then

$$
\hat{Y}=\hat{\alpha}+\hat{\beta}_{1}=47,099+21,160=68,259
$$

"The average price of houses with a driveway but no rec room is $\$ 68,259$ ".
3. If $D_{1}=0$ and $D_{2}=1$, then

$$
\hat{Y}=\hat{\alpha}+\hat{\beta}_{2}=47,099+16,024=63,123
$$

"The average price of houses with a rec room but no driveway is $\$ 63,123$ ".
4. If $D_{1}=0$ and $D_{2}=0$, then

$$
\hat{Y}=\hat{\alpha}=47,099
$$

"The average price of houses with no driveway and no rec room is $\$ 47,099$ ".

Multiple Regression with Dummy and non-Dummy Explanatory Variables

$$
Y=\alpha+\beta_{1} D+\beta_{2} X+e
$$

## Example: Explaining house prices (continued)

Regress $\mathbf{Y}=$ house price on $\mathbf{D}=$ air conditioning dummy and $X=$ lot size.

OLS estimates:

$$
\begin{gathered}
\hat{\alpha}=32,693 \\
\hat{\beta}_{1}=20,175 \\
\hat{\beta}_{2}=5.64
\end{gathered}
$$

Verbal ways of expressing OLS results:

- "An extra square foot of lot size will tend to add $\$ 5.64$ onto the price of a house" (Note: no ceteris paribus qualifications to statement since marginal effect is same for houses with and without air conditioners)
- "Houses with air conditioners tend to be worth $\mathbf{\$ 2 0 , 1 7 5}$ more than houses with no air conditioners, ceteris paribus" (Note: Here we do have ceteris paribus qualification)
- "If we consider houses with similar lot sizes, those with air conditioners tend to be worth an extra $\mathbf{\$ 2 0 , 1 7 5 "}$


## Example: Explaining house prices

(continued)

- For houses with an air conditioner $\mathbf{D}=1$ and

$$
\hat{\mathbf{Y}}_{\mathrm{i}}=52,868+5.64 X_{i}
$$

- For houses without an air conditioner $\mathbf{D}=0$ and

$$
\hat{\mathbf{Y}}_{i}=32,693+5.64 X i
$$

- Two different regression lines depending on whether the house has an air conditioner or not.
- Two lines have different intercepts but same slope (i.e. same marginal effect)

Another House Price Regression
$\mathbf{Y}=\alpha+\beta_{1} \mathbf{D}_{1}+\beta_{2} \mathbf{D}_{2}+\beta_{3} \mathbf{X}_{1}+\beta_{4} \mathbf{X}_{2}+\mathrm{e}$.

- Regress $Y=$ house price on $D_{1}=$ dummy variable for driveway, $\mathrm{D}_{2}=$ dummy variable for rec room, $X_{1}=$ lot size and $X_{2}=$ number of bedrooms
- OLS estimates:

$$
\begin{gathered}
\hat{\alpha}=-2736 \\
\hat{\beta}_{1}=12,598 \\
\hat{\beta}_{2}=10,969 \\
\hat{\beta}_{3}=5.197 \\
\hat{\beta}_{4}=10,562
\end{gathered}
$$

1. If $D_{1}=1$ and $D_{2}=1$, then

$$
\hat{Y}=20,831+5.197 X_{1}+10,562 X_{2}
$$

This is the regression line for houses with a driveway and rec room.
2. If $D_{1}=1$ and $D_{2}=0$, then

$$
\hat{Y}=9,862+5.197 \times X_{1}+10,562 \times X_{2}
$$

This is the regression line for houses with a driveway but no rec room.
3. If $D_{1}=0$ and $D_{2}=1$, then

$$
\hat{Y}=8,233+5.197 X_{1}+10,562 X_{2}
$$

This is the regression line for houses with a rec room but no driveway.
4. If $D_{1}=0$ and $D_{2}=0$, then

$$
\hat{Y}=-2,736+5.197 X_{1}+10,562 X_{2}
$$

This is the regression line for houses with no driveway and no rec room.

## Interacting Dummy and Non-Dummy Variables

$$
\mathbf{Y}=\alpha+\beta_{1} \mathbf{D}+\beta_{2} \mathrm{X}+\beta_{3} \mathrm{Z}+\mathrm{e}
$$

where

$$
\mathbf{Z}=\mathbf{D} \times \mathbf{X}
$$

- $Z$ is either 0 (for observations with $D=0$ ) or $X$ (for observations with $\mathrm{D}=1$ )
- If $\mathrm{D}=1$ then $\hat{Y}=\left(\hat{\alpha}+\hat{\boldsymbol{\beta}}_{1}\right)+\left(\hat{\boldsymbol{\beta}}_{2}+\hat{\boldsymbol{\beta}}_{3}\right) X$.
- If $\mathbf{D}=\mathbf{0}$, then $\hat{Y}=\hat{\alpha}+\hat{\beta}_{2} X$.
- Two different regression lines corresponding to $D=0$ and $D=1$ exist and have different intercepts and slopes.
- The marginal effect of $X$ on $Y$ is different for $D=0$ and $D=1$.


## Yet Another House Price Example

- Regress $Y=$ house price on $\mathbf{D}=$ air conditioner dummy, $X=$ lot size and $Z=D \times X$
- OLS estimates:

$$
\begin{gathered}
\hat{\alpha}=35,684 \\
\hat{\beta}_{1}=7,613 \\
\hat{\beta}_{2}=5.02 \\
\hat{\beta}_{3}=2.25 .
\end{gathered}
$$

- The marginal effect of lot size on housing is 7.27 for houses with air conditioners and only $\$ 5.02$ for houses without.
- Increasing lot size will tend to add more to the value of a house if it has an air conditioner than if it does not.

Working with Dummy Dependent Variables

## Example:

Dependent variable is a transport choice.
1 = "Yes I take my car to work"
$0=$ "No I do not take my car to work"

- We will not discuss this case.
- Note only the following points:

1. There are some problems with OLS estimation. But OLS estimation might be adequate in many cases.
2. Better estimation methods are "Logit" and "Probit". Excel cannot easily do these, so you will need a different software package.

## Chapter 9: Univariate Time Series Analysis

## Example: log of US personal income from 1954Q1 through 1994Q4.

- In previous chapter we discussed distributed lag models. These can be misleading if:

1. The dependent variable $Y_{t}$ depends on lags of the dependent variable as well, possibly, as $\mathrm{X}_{\mathrm{t}}$, $\mathbf{X}_{\mathrm{t}-1}, \ldots, \mathbf{X}_{\mathrm{t}-\mathrm{q}}$.
2. The variables are nonstationary.

- In this chapter and the next, we develop tools for dealing with both issues and define what we mean by "nonstationary".
- To simplify the analysis, focus solely on one time series, Y. Hence, univariate time series analysis.
- It is important to understand the properties of each individual series before proceeding to regression modelling involving several series.


## Example: US personal income (cont.)

- Note trend behaviour of personal income series.
- Many macroeconomic time series exhibit such trends.

See Figure 9.1

Aside on logs

- It is common to take the natural logarithm of time series which are growing over time (i.e. work with $\ln (\mathrm{Y})$ instead of Y$)$. Why?
- A time series graph of $\ln (\mathrm{Y})$ will often approximate a straight line.
- In regressions with logged variables coefficients can be interpreted as elasticities.
- $\ln \left(\mathrm{Y}_{\mathrm{t}}\right)-\ln \left(\mathrm{Y}_{\mathrm{t}-1}\right)$ is (approximately) the percentage change in $Y$ between period $t-1$ and $t$.


## Differencing

$$
\Delta \mathbf{Y}_{\mathbf{t}}=\mathbf{Y}_{\mathbf{t}}-\mathbf{Y}_{\mathrm{t}-1}
$$

- $\Delta \mathbf{Y}_{\mathbf{t}}$ measures the change (or growth) in $\mathbf{Y}$ between periods $\mathbf{t - 1}$ and $t$.
- If $Y_{t}$ is the $\log$ of a variable, then $\Delta Y_{t}$ is the percentage change.
- $\Delta \mathbf{Y}_{t}$ is the difference of $\mathbf{Y}$ (or first difference).
- $\Delta \mathbf{Y}_{\mathrm{t}}$ is often called "delta Y ".


## Example: US personal income (cont.)

## The Autocorrelation Function

## See Figure 9.2.

- $\Delta \mathbf{Y}=\%$ change in personal income
- Not trending, very erratic.
- The differences of many macroeconomic time series have such properties.


## The Autocorrelation Function: Notation

- $r_{1}=$ correlation between $Y$ and $Y$ lagged one period.
- $r_{p}=$ correlation between $Y$ and $Y$ lagged $p$ periods.
- Autocorrelation function treats $\mathbf{r}_{\mathbf{p}}$ as a function of $p$.
- Correlation between $Y$ and lags of itself shed important light of the properties of $Y$.
- Relates to the idea of a trend (discussed above) and nonstationarity (not discussed yet).

Example: Y = US personal income

- Correlation between $Y_{t}$ and $Y_{t-1}$ is . $\mathbf{9 9 9 7 1 6}$ !
- Correlation between $\Delta Y_{t}$ and $\Delta Y_{t-1}$ is -. 00235 .
- These are autocorrelations (i.e. correlations between a variable and lags of itself).


## Example: US personal income (cont.)

Autocorrelation functions of $Y$ and $\Delta Y$

| Lag length (p) | Personal <br> Income | Change in <br> Pers. Income |
| :---: | :---: | :---: |
| 1 | .9997 | -.0100 |
| 2 | .9993 | .0121 |
| 3 | . .9990 | .1341 |
| 4 | .9986 | .0082 |
| 5 | .9983 | -.1562 |
| 6 | .9980 | .0611 |
| 7 | .9978 | -.0350 |
| 8 | . .9975 | -.0655 |
| 9 | .9974 | .0745 |
| 10 | .9972 | . .1488 |
| 11 | .9969 | .0330 |
| 12 | .9966 | .0363 |

- $Y$ is highly correlated with lags of itself, but the change in $Y$ is not.
- Information could also be presented on bar charts. See Figures 9.3 and 9.4.


## Autocorrelation: Intuition

## The Autoregressive Model

- $Y$ is highly correlated over time. $\Delta Y$ does not exhibit this property.
- If you knew past values of personal income, you could make a very good estimate of what personal income was this quarter. However, knowing past values of the change in personal income will not help you predict the change in personal income this quarter.
- $Y$ "remembers the past". $\Delta Y$ does not.
- $Y$ is a nonstationary series while $\Delta Y$ is stationary. (Note: These words not formally defined yet.)


## Nonstationary vs. Stationary Time Series

- Formal definitions require difficult statistical theory. Some intuition will have to suffice.
- "Nonstationary" means "anything which is not stationary".
- Focus on a case of great empirical relevance: unit root nonstationarity.
- Previous discussion has focussed on graphs and correlations, now we go on to regression.
- Autoregressive model of order 1 is written as AR(1) and given by:

$$
\mathbf{Y}_{\mathbf{t}}=\alpha+\phi \mathbf{Y}_{\mathrm{t}-1}+\mathbf{e}_{\mathrm{t}}
$$

- Figures 9.5, 9.6 and 9.7 indicate the types of behaviour that this model can generate.
- $\phi=1$ generates trending behaviour typical of macro time series.
- $\phi=0$ looks more like change in macro time series.


## Ways of Thinking about Whether $\mathbf{Y}$ is Stationary or has a Unit Root

1. If $\phi=1$, then $Y$ has a unit root. If $|\phi|<1$ then $Y$ is stationary.
2. If $Y$ has a unit root then its autocorrelations will be near one and will not drop much as lag length increases.
3. If $Y$ has a unit root, then it will have a long memory. Stationary time series do not have long memory.
4. If $Y$ has a unit root then the series will exhibit trend behaviour.
5. If $Y$ has a unit root, then $\Delta Y$ will be stationary. Hence, series with unit roots are often referred to as difference stationary.

$$
\mathbf{Y}_{t}=\alpha+\phi \mathbf{Y}_{t-1}+\mathbf{e}_{t}
$$

- Can rewrite as:

$$
\begin{aligned}
& \Delta \mathbf{Y}_{\mathrm{t}}=\alpha+\rho \mathbf{Y}_{\mathrm{t}-1}+\mathbf{e}_{\mathrm{t}} \\
& \text { where } \rho=\phi-\mathbf{1}
\end{aligned}
$$

- If $\phi=1$ (unit root) then $\rho=0$ and:

$$
\Delta \mathbf{Y}_{\mathrm{t}}=\alpha+\mathbf{e}_{\mathrm{t}}
$$

- Intuition: if Y has a unit root, can work with differenced data --- differences are stationary.


## The AR(p) Model

- Coefficients in this alternative regression ( $\rho$, $\left.\gamma_{1}, \ldots, \gamma_{p-1}\right)$ are simple functions of $\phi_{1}, . ., \phi_{\mathrm{p}}$.
- Test if $\rho=0$ to see if a unit root is present.
- $-1<\phi<1$ is equivalent to $-2<\rho<0$. This is called the stationarity condition.


## Aside: The Random Walk Model:

$$
\mathbf{Y}_{t}=\mathbf{Y}_{t-1}+\mathbf{e}_{\mathrm{t}}
$$

- This is thought to hold for many financial variables such as stock prices, exchange rates.
- Intuition: Changes in Y are unpredictable, so no arbitrage opportunities for investors.
- AR(p) model:

$$
\boldsymbol{Y}_{t}=\alpha+\phi_{1} \mathbf{Y}_{t-1}+\ldots+\boldsymbol{\phi}_{p} \mathbf{Y}_{t-p}+\boldsymbol{e}_{t},
$$

- Properties similar to the AR(1) model.
- Alternative way of writing AR(p) model:

$$
\Delta \boldsymbol{Y}_{t}=\alpha+\rho \boldsymbol{Y}_{t-1}+\gamma_{t} \Delta \boldsymbol{Y}_{t-1}+\ldots+\gamma_{p-1} \Delta \boldsymbol{Y}_{t-p+1}+\boldsymbol{e}_{t}
$$

## Extensions of the AR(1) Model

- $A R(p)$ is in the form of a regression model.
- $\rho=0$ implies that the time series $Y$ contains a unit root (and $-2<\rho<0$ indicates stationarity).
- If a time series contains a unit root then a regression model involving only $\Delta Y$ is appropriate (i.e. if $\rho=0$ then the term $Y_{t-1}$ will drop out of the equation).
- "If a unit root is present, then you can difference the data to induce stationarity."


## More Extensions: Adding a

## Summary

Deterministic Trend

- Consider the following model:

$$
Y_{t}=\alpha+\delta t+e_{t} .
$$

- The term $\delta \boldsymbol{t}$ is a deterministic trend since it is an exact (i.e. deterministic) function of time.
- Unit root series contain a so-called stochastic trend.
- Combine with the AR(1) model to obtain:

$$
Y_{t}=\alpha+\phi Y_{t-1}+\delta t+e_{t}
$$

- Can generate behaviour that looks similar to unit root behaviour even if $|\phi|<1$. (i.e. even if they are stationary).
- See Figure 9.8.


## AR(p) with Deterministic Trend Model

- Most general model we use:

$$
\Delta \boldsymbol{Y}_{t}=\alpha+\rho \boldsymbol{Y}_{t-1}+\gamma_{t} \Delta \boldsymbol{Y}_{t-1}+\ldots+\gamma_{p} \Delta \boldsymbol{Y}_{t-p+1}+\delta \boldsymbol{t}+\boldsymbol{e}_{t} .
$$

- Why work with this form of the model?

1. A unit root is present if $\rho=0$. Easy to test.
2. The specification is less likely to run into multicollinearity problems. Remember: in macroeconomics we often find $Y$ is highly correlated with lags of itself but $\Delta \mathbf{Y}$ is not.

- The nonstationary time series variables on which we focus are those containing a unit root. These series contain a stochastic trend. If we difference these time series, the resulting time series will be stationary. For this reason, they are also called difference stationary.
- The stationary time series on which we focus have $-2<\rho<0$. But these series may exhibit trend behaviour through the incorporation of a deterministic trend. If this occurs, they are also called trend stationary.
- OLS can be done in usual way.

Example: Y = US personal income

- $\Delta \mathbf{Y}$ is the dependent variable in the regression below.


## AR(4) with Deterministic Trend Model

|  | Coeff. | St. <br> Error | t-Stat | P-val | Lower <br> $\mathbf{9 5 \%}$ | Upper <br> $\mathbf{9 5 \%}$ |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: |
| Inter. | .138 | .108 | 1.279 | .203 | -.075 | .351 |
| $\mathbf{Y}_{\mathbf{t}-\mathbf{1}}$ | -.018 | .015 | -1.190 | .236 | -.049 | .012 |
| $\Delta \mathbf{Y}_{\mathbf{t}-1}$ | -.017 | .081 | -.217 | .829 | -.177 | .142 |
| $\Delta \mathbf{Y}_{\mathbf{t}-2}$ | .014 | .081 | .172 | .863 | -.145 | .173 |
| $\Delta \mathbf{Y}_{\mathbf{t}-3}$ | . $\mathbf{. 1 3 0}$ | $\mathbf{. 0 8 0}$ | $\mathbf{1 . 6 2 7}$ | .106 | -.028 | .288 |
| time | . $\mathbf{. 0 0 0 1 2}$ | . $\mathbf{0 0 0 1 2}$ | .955 | .341 | -.00013 | .00037 |

## Testing in AR(p) with Deterministic Trend Model

- For everything except $\rho$, testing can be done in usual way using $t$-statistics and $P$-values.
- Hence, can use standard tests to decide whether to include deterministic trend.


## Lag length selection

- A common practice: begin with an AR(p) model and look to see if the last coefficient, $\gamma_{p}$ is significant. If not, estimate an $\operatorname{AR}(\mathbf{p}-1)$ model and see if $\gamma_{\mathrm{p}-1}$ is significant. If not, estimate an $\operatorname{AR}(p-2)$, etc.


## Testing for a Unit Root

- You might think you can test $\rho=0$ in the same way (i.e. look at $P$-value and, if it is less than . 05 , reject the unit root hypothesis, if not accept the unit root).
- THIS IS INCORRECT!
- Justification: Difficult statistics.
- Essentially: Excel gets the t-statistic correct, but the $P$-value is wrong.
- A correct test is the Dickey-Fuller Test, which uses the $t$-statistic and compares it to a critical value.

Example: Y = US personal income

- Sequential testing strategy leads us to drop the deterministic trend and go all the way back to a model with one lag, an AR(1).
- $\Delta Y$ is the dependent variable in the regression.

|  | Coeff. | St. <br> Error | t-Stat | P-val | Lower <br> 95\% | Upper <br> $\mathbf{9 5 \%}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Inter. | $\mathbf{. 0 3 9}$ | $\mathbf{. 0 1 4}$ | 2.682 | $\mathbf{. 0 0 8}$ | $\mathbf{. 0 1 0}$ | $\mathbf{. 0 6 7}$ |
| $\mathbf{Y}_{\mathbf{t}-1}$ | -.004 | $\mathbf{. 0 0 2}$ | -2.130 | $\mathbf{. 0 3 5}$ | -.0077 | $\mathbf{- . 0 0 0 3}$ |

## Practical Advice on Unit Root Testing

- Learn more econometrics using another textbook.
- Use a computer software package which is suitable for time series.
- Use the following rough rule of thumb which should be okay if sample size is moderately large (e.g. T>50).


## Testing for a Unit Root: An Approximate Strategy

1. Use the sequential testing strategy outlined above to estimate the AR(p) with deterministic trend model. Record the t-stat corresponding to $\rho$ (i.e. the coefficient on $\mathbf{Y}_{\mathrm{t}-1}$ ).
2. If the final version of your model includes a deterministic trend, the Dickey-Fuller critical value is approximately -3.45 . If the $t$-stat on $\rho$ is more negative than -3.45 , reject the unit root hypothesis and conclude that the series is stationary. Otherwise, conclude that the series has a unit root.
3. If the final version of your model does not include a deterministic trend, the DickeyFuller critical value is approximately -2.89. If the $t$-stat on $\rho$ is more negative than this, reject the unit root hypothesis and conclude that the series is stationary. Otherwise, conclude that the series has a unit root.

Example: Y = US personal income

- Sequential testing strategy leads from:

$$
\Delta \boldsymbol{Y}_{t}=\alpha+\rho \boldsymbol{Y}_{t-1}+\gamma_{t} \Delta \boldsymbol{Y}_{t-1}+\ldots+\gamma_{p} \Delta \boldsymbol{Y}_{t-p+1}+\delta t+\boldsymbol{e}_{t}
$$

to

$$
\Delta \mathbf{Y}_{t}=\alpha+\rho \mathbf{Y}_{t-1}+\mathbf{e}_{\mathbf{t}}
$$

|  | Coeff. | St. <br> Error | t-Stat | P-val | Lower <br> 95\% | Upper <br> $\mathbf{9 5 \%}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Inter. | $\mathbf{. 0 3 9}$ | $\mathbf{. 0 1 4}$ | 2.682 | .008 | $\mathbf{. 0 1 0}$ | $\mathbf{. 0 6 7}$ |
| $\mathbf{Y}_{\mathbf{t}-1}$ | -.004 | .002 | -2.130 | .035 | -.0077 | -.0003 |

- t-stat on $\rho$ is -2.13, which is not more negative than -2.89 . Hence, we can accept the hypothesis that personal income does contain a unit root.


## College Town

FCI owns 10 apartment buildings in a college town, which it rents exclusively to students. Each apartment building contains 100 rental units, but the owner is having cash flow problems due to an average vacancy rate of nearly 50 percent. The apartments in each building have comparable floor plans, but some buildings are closer to campus than others. The owner of FCI has data from last year on the number of apartments rented, the rental price (in dollars), and the amount spent on advertising (in hundreds of dollars) at each of the 10 apartments. These data, along with the distance (in miles) from each apartment building to campus, are presented in rows 1 through 11 of Table 3-9. The owner regressed the quantity demanded of apartments on price, advertising, and distance. The results of the regression are reported in rows 16 through 35 of Table 3-9. What is the estimated demand function for FCI's rental units? If FCI raised rents at one complex by $\$ 100$, what would you expect to happen to the number of units rented? If FCI raised rents at an average apartment building, what would happen to FCI's total revenues? What inferences should be drawn from this analysis?

Table 3-9 Input and Output from a Multiple Regression

|  | A | B | C | D | E | F | G |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | Observation | Guantity | Price | Advertising | Distance |  |  |
| $\mathbf{2}$ | 1 | 28 | 250 | 11 | 12 |  |  |
| $\mathbf{3}$ | 2 | 69 | 400 | 24 | 6 |  |  |
| $\mathbf{4}$ | 3 | 43 | 450 | 15 | 5 |  |  |
| $\mathbf{5}$ | 4 | 32 | 550 | 31 | 7 |  |  |
| $\mathbf{6}$ | 5 | 42 | 575 | 34 | 4 |  |  |
| $\mathbf{7}$ | 6 | 72 | 375 | 22 | 2 |  |  |
| $\mathbf{8}$ | 7 | 66 | 375 | 12 | 5 |  |  |
| $\mathbf{9}$ | 8 | 49 | 450 | 24 | 7 |  |  |
| $\mathbf{1 0}$ | 9 | 70 | 400 | 22 | 4 |  |  |
| $\mathbf{1 1}$ | 10 | 60 | 375 | 10 | 5 |  |  |
| $\mathbf{1 2}$ | Average | 53.10 | 420.00 | 20.50 | 5.70 |  |  |
| $\mathbf{1 3}$ |  |  |  |  |  |  |  |
| $\mathbf{1 4}$ |  |  |  |  |  |  |  |
| $\mathbf{1 5}$ |  |  |  |  |  |  |  |
| $\mathbf{1 6}$ | Regression Statistics |  |  |  |  |  |  |
| $\mathbf{1 7}$ |  |  |  |  |  |  |  |
| $\mathbf{1 8}$ | Muliple $R$ |  |  |  |  |  |  |
| $\mathbf{1 9}$ | RSquare | 0.89 |  |  |  |  |  |
| $\mathbf{2 0}$ | Adjusted RSquare | 0.79 |  |  |  |  |  |
| $\mathbf{2 1}$ | Standard Error | 9.18 |  |  |  |  |  |
| $\mathbf{2 2}$ | Observations | 10.00 |  |  |  |  |  |
| $\mathbf{2 3}$ |  |  |  |  |  |  |  |

Quelle: Baye, Michael: Managerial Economics and Business Strategy, Mc Graw 2003, p 101

Analyse the data in Excel:

- Describe the relation between the quantity and the independent variables in bivariate correlations and regressions. What do we learn?
- Continue the multiple regression (already started above).
o Which are the numerical results?
o What do we learn? Interpret the results (for example answering the questions above); what is your recommendation for FCI ?


[^0]:    6

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